PhilMath Intersem 8. 2017

The PhilMath Intersem (PhilMath Intersem 8) will convene for its eighth annual meetings during June 2017. The theme of this year's seminar will be the infinite and its roles in the history of mathematics. Historical, philosophical, logical and mathematical issues will be considered.

All meetings will take place on the Rive Gauche campus of the University of Paris 7-Diderot, in the salle Klimt (room 366A) of the Condorcet Building.

Schedule

Meeting 1: Tuesday, June 6th

• **14h00: Richard Arthur** (Dept. of Philosophy, McMaster University)

"Mathematics and the Leibnizian actual infinite"
It is well known that Leibniz advocated the actual infinite, but rejected infinite number and infinite collections as violating the part-whole axiom. He described this conception of the actual infinite using the Scholastic term “syncategorematic”. An infinite aggregate or plurality, understood syncategorematically, is one with so many terms that, no matter how many are enumerated, there are actually more. It differs from the potential infinite of traditional mathematics in upholding the actual infinitude of terms, but it denies that they form an infinite whole or collection. It is thus incompatible with the actually infinite sets proposed by Cantor—even including the set of natural numbers endorsed by the intuitionists. Nevertheless, Leibniz is able to give an account of infinite aggregates, such as the actual infinity of terms in an infinite series, where he gives a perfectly rigorous account of what it means for a converging series to have an infinite sum, without there being a collection of the terms that are summed (it agrees with the modern definition in terms of a limit of partial sums, without invoking the concept of a limit). In this talk I explore some of the implications of adopting such a view of the actual infinite for the foundations of mathematics.

• **16h00: Ohad Nachtomy** (Dept. of Philosophy, Bar-Ilan University)

"On Living Mirrors and Mites: Leibniz and Pascal on Infinity and Life"
My focus will be a dense and rich text in which Leibniz comments on fragment 22 of Pascal’s Pensées in the Port Royal Edition (currently indexed Lafuma 199). The text was published by Gaston Grua under the title Double infinité chez Pascal et Monade and was dated by Grua to sometimes after 1695. It has received a fair amount of commentary in French but none in English. In this text, Leibniz responds to Pascal’s employment of the infinitely large and infinitely small and, more specifically, to the way he uses infinity to describe living beings through the example of a mite (ciron). In contrast to Pascal’s mite, Leibniz is employing the image of a living mirror (miroir vivant). I use these images to draw some similarities but especially contrasts between Leibniz and Pascal’s employment of infinity to depict the nature of living beings. In sections 2-5, I present and analyze the text, focusing on the major differences in Pascal's and Leibniz’s uses of infinity. In sections 6-7, I account for the late appearance of this text by looking at the development of Leibniz’s view of infinity and the role it played in his definition of living beings. I argue that, in spite of superficial similarities, Leibniz’s use of infinity to define living beings stands in stark contrast to Pascal’s use of infinity in that it stresses unity and harmony rather than divisibility and disparity. Leibniz’s use of

infinity through the notion of a living mirror suggests that each individual being, no matter how minute, forms an integral part of a well-connected and harmonious system. Whereas Pascal uses infinity to highlight our alienation from and incomprehension of the world, in Leibniz, infinity serves as a mark of unity, connectedness, and, one might say, a sense of belonging.

Meeting 2: Thursday, June 8th

• **14h00: Pieter Sjørd Hasper** (Dept. of Philosophy, Indiana University-Bloomington)

"Physics and mathematics in Aristotle's Account of Infinity"
Aristotle is famous for denying that there is anything infinite in reality, both in physical reality and in the reality described by mathematics. On the other hand, he is well aware that we need some infinities in physics and especially in mathematics. In order to reconcile these two claims, he introduces the notion of the potential infinite: there are infinitely many possibilities, but in actuality there will be nothing infinite. It has been argued that that this does not suffice, notably in the case of the infinite past.
In this paper I will first argue that the underlying common understanding of Aristotle's account of the potential infinite is incorrect and that we should understand it in a weaker way, which will also fit the evidence better. On this alternative understanding one can see that the infinity of the past poses no problem, but also that Aristotle has a better way of accommodating the infinities required by mathematics (of infinitely extendible lines and of infinitely many numbers) than is usually supposed. Some striking arguments against the infinite size of the universe can also be understood better.

• **16h00: Ansten Klev** (Institute of Philosophy, Academy of Sciences of the Czech Republic)

"Theorem 66: Dedekind's route to infinity"
Theorem 66 of Dedekind's *Was sind und was sollen die Zahlen?* (1888) asserts that there exists an infinite set. The proof Dedekind offers of this theorem may well be described as a singularity of an otherwise sober and rigorous development of arithmetic: it aims to show that *meine Gedankenwelt* is an infinite set in Dedekind's sense, namely a set that can be mapped injectively into a proper subset of itself. I first wish to elucidate Dedekind's proof from various angles: the content of its premisses; connections to the works of other philosophers and mathematicians; and its reception. I then want to consider to what extent the proof can be regarded as a `logical proof of existence', as Dedekind himself describes it in his famous letter to Keferstein. That the proof may be classified as logical is presupposed by the characterization of Dedekind as a logicist about arithmetic.

Meeting 3: Tuesday, June 13th

• **16h00: Pascal Crozet** (SPHERE, Université Paris 7-Diderot)
"Figures of infinity in Arabic mathematics"
Between Antiquity and the Classical Age, Arabic mathematics met the question of the infinite in several fields and traditions of research: development of the Archimedean tradition for the measurement of surfaces and volumes; problem of the asymptote; anthyphairetic definition of proportions; arithmetical treatment of irrational magnitudes, etc. We will give a broad overview of these issues.

Meeting 4: !! Wednesday !!, June 14th

• **16h00: José Ferreirós** (Lógica y Filosofía de la Ciencia, Universidad de Sevilla)

"Cantor, the actual/potential divide, and the impact of the paradoxes"
The question of who discovered the set-theoretic paradoxes, and exactly when, has been a matter of contention. Some authors point to Russell in the 1900s, some to Burali-Forti in 1897, some to Cantor himself but with disagreements as to date.
In an interesting reconstruction of a possible pathway to the paradoxes, Bill Tait (2000) defended that Cantor in fact discovered them in the 1880s within the context of his *Grundlagen* published in 1883. There is one letter of the famous mathematician, and an obscure passage in the *Grundlagen*, that indeed support that reading. Yet a significant amount of textual evidence contradicts that perspective, suggesting that Cantor was still unaware of the paradoxes as late as the early 1890s.
We shall reconsider the matter, showing that it is most likely that Cantor discovered the paradoxes of transfinite numbers in 1896. Tait's rational reconstruction will thus be regarded as disproved by historical evidence. As will become clear, the discovery triggered an interesting shift in Cantor's thinking from an actualist to a potentialist understanding of the absolutely infinite.

Meeting 5: Tuesday, June 20th
• **14h00: John Stillwell** (Dept. of Mathematics, University of San Francisco)
"Infinity in the History of Mathematics"
Infinity first appeared significantly in the mathematics of ancient Greece, with the discovery of irrational numbers and the theory of area and volume. The reaction at the time was avoid infinity as far as possible through Eudoxus' theory of proportions and method of exhaustion, which replaced the actual infinite by the arbitrary finite. This led to the belief (which persisted until the 19th century) that mathematics could manage without the actual infinite, though the work of Archimedes suggested that infinity was nevertheless a powerful method of discovery.
The next significant encounter with infinity occurred with the development of calculus in the 17th century. Some of the 17th-century uses of infinity were like those of the Greeks, and hence could be justified by the method of exhaustion. But, increasingly, the method of exhaustion was abandoned in favor of calculation with « infinitesimals." Infinitesimals were efficient, but "infinitely small," and hence also under suspicion.
The concept of infinity came under fresh scrutiny in the 19th century when Gauss's proofs of the fundamental theorem of algebra raised questions about the nature of continuous functions. Bolzano in 1817 traced the difficulty to a question about the real numbers (completeness) -- a question that could not be answered until the real numbers were precisely defined. Suitable definitions (by Dedekind and others) appeared in the second half of the 19th century, quickly followed by Cantor's discovery that the reals form an uncountable set -- necessarily an actual infinity -- and that in fact there are many levels of infinity.
With this discovery, the old question of whether to accept actual infinity became much more complicated. We should ask, rather: how much infinity should be accepted? In the 20th century logicians made a lot of progress on this question, by finding out how much needs to be assumed about infinity in order to answer natural questions in mainstream mathematics.

• **16h00: Stephen Simpson** (Dept. of Mathematics, Vanderbilt University)

"Actual infinity, potential infinity, objectivity, and reverse mathematics"
In 1926 Hilbert proposed a sweeping program whereby the entire panorama of higher mathematical abstractions would be justified objectively and logically, in terms of finite processes. But then in 1931 Kurt Gödel published his famous incompleteness theorems, leading to an era of confusion and skepticism. In this talk I show how modern foundational research has opened a new path toward objectivity and optimism in mathematics.

Meeting 6: Thursday, June 22nd

• **16h00: Cédric Vergnerie** (SPHERE, Université Paris 7-Diderot)
"Algebra without the "Fiction of the roots": Kronecker and Sturm's theorem"
For the better part of the second half of the nineteenth century, Leopold Kronecker gave a course on the theory of algebraic equations, which represents his Algebra lectures. In this *Vorlesungen über der Theorie der algebraischen Gleichungen*, he asked Algebra to be "as far as possible independent from all the fictions about the roots of equations" ("*Die Resultate der Algebra sind nach Möglichkeit von allen Fiktionen über die Wurzeln der Gleichungen unabhängig zu machen*").
We will show how Kronecker, in the context of the generalization of the Sturm's theorem, has dealt with the notion of continuity and how he tried to remove the whole concept of root.

Meeting 7: Tuesday, June 27th

• **16h00: Joao Cortese** (Université de Paris & Universidada de São Paolo)
""A nothingness with regard to the infinite": Pascal and the relational and absolute aspects of infinity"
Aristotle's distinction between actual and potential infinity is a classical one, which cannot be confused with the medieval distinction between categorematic and syncategorematic terms. Besides, in Seventeenth Century one of the main questions on the foundations of the methods of indivisibles was whether indivisibles were homogeneous or heterogeneous to their corresponding magnitudes (cf. *Elements*, V, definitions 3 and 4). Related to all these distinctions, one can consider infinity as a relational or as an absolute entity.
In this talk, I will show that in order to make sense of Pascal's reflections on "indivisibles", "nothingness" and "infinity", one should make usage of an absolute-relational kind of distinction. On the one hand, in the Lettres de A. Dettonville, Pascal makes divisions of a magnitude in an "indefinite" number of equal parts, making the difference "smaller than any given magnitude". On the other hand, in the De l'esprit géométrique, he says clearly that no division from a magnitude can be made such that an "indivisible" is found, because they are heterogeneous one another. I will argue that the defense of these two positions is not contradictory if we accept that these terms are used by Pascal sometimes in a relational meaning and sometimes in an absolute one. In order to discuss this, I will focus on some case studies of Pascal's mathematical practice on the methods of indivisibles and in his reflections about infinity in *De l'esprit géométrique* and in the *Pensées*.

Meeting 8: Thursday, June 29th
• **14h00: Paolo Mancosu** (Dept. of Philosophy, University of California-Berkeley)
"How should we determine the size of infinite sets?"
In two recent articles (Mancosu 2009 and 2015), I have addressed the mathematical and philosophical issues connected to the problem of how one should generalize counting from finite to infinite sets. Historically, several conflicting intuitions have been proposed based on different intuitive criteria such as preservation of the part-whole relation (Bolzano), assignment of the same size to all infinite sets (function Num by Peano), and one-to-one correspondence (Cantor, Frege). The Cantorian solution in terms of one-to-one correspondence has been so successful that no mathematical alternatives seemed possible for a long time. Recent developments in mathematics (theory of numerosities) have broadened the range of conceptual possibilities and have led to important philosophical consequences concerning the inevitability of the Cantorian solution (Gödel) and the status of alternatives to Hume's Principle in neo-logicism. In my talk I give an overview of my work in this area and point to several problems that still call for investigation. I will also, time

permitting, connect this material to similar issues that emerge in generalizing probability from finite to infinite settings (De Finetti's lottery etc).

• **16h00: Philip Welch** (Dept. of Mathematics, University of Bristol)
"Reflecting on Absolute Infinity"
This talk is based on a paper with Leon Horsten (of the same title) in a recent J.of Phil. article and a talk in the EFI series.
We reflect on the ineffability of the Cantorian Absolute. If this is done in the style of Levy, Montague in first order manner, or Bernays using second or higher order methods this has only resulted in principles that can justify large cardinals that are `intra-constructible', that is they do not contradict the assumption that V, the universe of sets of mathematical discourse, is Gödel's universe of constructible sets, L. Peter Koellner has advanced reasons that this style of reflection will only have this rather limited strength. However set theorists would dearly like to have much stronger axioms of infinity. We propose a widened Global Reflection Principle that is based on a view of sets and Cantorian absolute infinities that delivers a proper class of Woodin cardinals (and more). A mereological view of classes is used to differentiate between sets and classes. Once allied to a wider view of structural reflection, stronger conclusions are possible.

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