The Eighth French Philosophy of Mathematics Workshop

CAN WE HAVE MATHEMATICAL UNDERSTANDING OF PHYSICAL PHENOMENA?

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My aim: to argue that in the cases of alleged mathematical explanations of physical phenomena discussed in the literature, mathematics is conveying understanding without playing an explanatory part.

- four examples and the reasons why they cannot be taken as genuine cases of mathematical explanation
- the relation between understanding and explanation: can we have one without the other?
- a new account - one that shows how mathematics can convey understanding in these cases without having explanatory value.

We can distinguish between two camps in the recent discussion about the role of mathematics in scientific explanations:

1. those that argue that mathematics can play an explanatory part in explanations of physical phenomena: Steiner (1978), Colyvan (2001), Baker (2005, 2009, 2012), Batterman (2010), Lyon (2012), Lange (2013), and Pincock (2015)
2. those that consider that mathematics doesn't play an explanatory role in science: Melia (2002), Bangu (2008), Daly and Langford (2009), Saatsi (2011), Rizza (2011), and Raz (2013, 2016)

My position: I agree with both camps. On the one hand I consider that mathematics doesn't play an explanatory part in science; on the other hand I do believe that it contributes to our understanding. There is an apparent tension here. After all, isn't understanding inextricably linked with explanations? So, how can one claim that mathematics can contribute to our understanding of physical phenomena without playing an explanatory role in science? If this is the case, my project is doomed from the start, that is why it is important to argue that, despite what many think, there are other ways of achieving understanding of physical phenomena than just by grasping a scientific explanation of it.

The explanandum: at a given time we discover on Earth two antipodal points with exactly the same temperature and barometric pressure. Why is that? Why are there any such antipodal points?

The Borsuk-Ulam theorem: for any continuous function $f: S^n \to R^n$ (from an n-sphere into Euclidean n-space) there exists $x \in S^n$ such that $f(x) = f(-x)$ (i.e. maps some pair of antipodal points to the same point)

For the case $n = 2$, this theorem can be interpreted as saying (assuming that the Earth is topologically equivalent to a sphere and that temperature and pressure change continuously across its surface) that there are always on Earth's surface, at a given time, antipodal points with the same temperature, i.e. exactly our explanandum.

Problems:

a. we are dealing with a prediction and not an explanation (Baker 2005)
b. we are not dealing with a phenomenon in need of an explanation (Baker 2005)
c. what proof of the theorem does Colyvan have in mind and what makes it explanatory?

   - "there is prima facie evidence against the proof in question being unexplanatory: namely, the connection between (the proofs of) the Borsuk-Ulam theorem and other fixed-point theorems (most notably, Brouwer's celebrated fixed-point theorem). These connections seem to give some plausibility to the proof in question being explanatory" (Baker and Colyvan 2011, p. 327).

d. what reason do we have to consider in the first place that there is some kind of relation between the temperature and pressure at these two remote points?

The explanandum: the prime-numbered length of the life-cycle of these insects (13 and 17 years, depending on the geographical area)

Mathematical theorem: the lowest common multiple of two numbers is maximal when the numbers are coprime.

The explanation: having a life-cycle period which minimizes intersection with other periods is evolutionarily advantageous because it either helps with avoiding predators or it diminishes the chances of hybridization with similar subspecies.

Structure:

(1) Having a life-cycle period which minimizes intersection with other (nearby/ lower) periods is evolutionarily advantageous. [biological 'law']

(2) Prime periods minimize intersection (compared to non-prime periods). [number theoretic theorem]

(3) Hence organisms with periodic life-cycles are likely to evolve periods that are prime. ['mixed' biological/mathematical law] (Baker 2005, p. 233).

Problems:

a. the property of being prime – of central importance in the explanandum – is not, in this case, and should not be interpreted as a property of some mathematical entities, but should be taken as having to do with the properties of time intervals corresponding to life-cycles (Davide Rizza 2011).

b. the explanandum is not in fact a pure physical phenomenon, but a mixture of physical and mathematical elements (Bangu 2008).

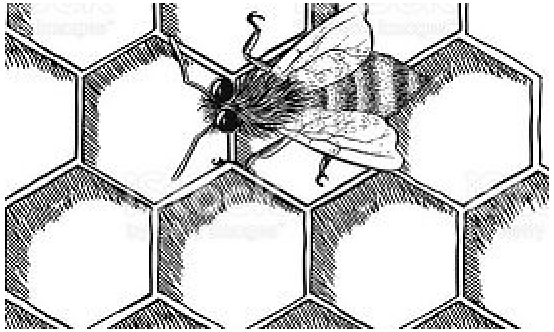
c. the role of numbers in this and similar cases is only to index a given duration (the durations of the life-cycles of the cicada) and so it plays no explanatory part. If we take this into account, we can give the following mathematics free paraphrase of the explanation: “the reason why the life-cycle of a given species of cicada is of the given particular number of units is that the duration which that number of units picks out minimizes the cicadas’ contact with other predatory species that inhabit the cicadas’ environment” (Daly and Langford 2009, p. 657).

The honeycomb structure

The explanandum: why bee's honeycombs have that particular shape

The honeycomb theorem: a hexagonal grid is the optimal way to divide into regions of equal area with least total perimeter a Euclidean plane.

The explanation: in order to win the natural selection fight, bees had to choose the most economic (in terms of labour and amount of wax used) way to build their honeycombs. As it is clear from the mathematical theorem presented above, from all the possible shapes, the hexagonal grid is the most economical in the relevant respects.



Problems:

a. the bees' honeycombs are in a nontrivial way three-dimensional and this makes the explanation at least incomplete; and, for the same reasons, the honeycomb theorem is probably irrelevant for the explanation of comb structures. So, "the explanation proposed by Lyon and Colyvan is an inadequate explanation of the structure of the bee's honeycomb" (Räz 2013, p. 354).

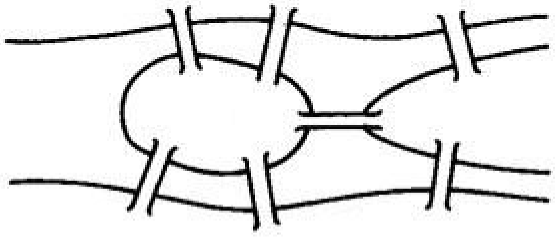
b. In Saatsi's view, the role of mathematics is that of representing a "crucial physical fact to which the full evolutionary explanation of the phenomenon then appeals" (Saatsi 2011, p. 146).

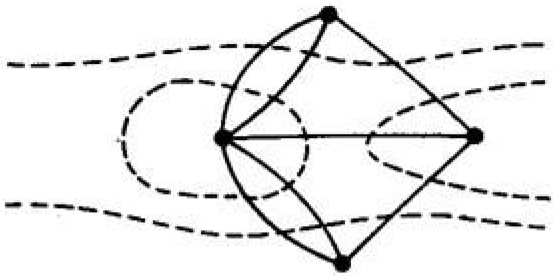
c. Lange takes both examples to be cases of causal, not mathematical, explanations. Regarding the explanation of the honeycomb structure, he says, for example, that it "works by describing the relevant features of the selection pressures that have historically been felt by honeybees, so it is an ordinary, causal explanation, not distinctively mathematical" (Lange 2013).

The bridges of Königsberg

The explanandum: Why no one can ever succeed in crossing each bridge in Königsberg only once and then return to the starting point?

The explanation: such a crossing is impossible because the geometrical graph of the bridges cannot have an Euler trail, i.e. a closed path that includes each edge of the graph only once. For such a trail to be possible, the graph must not include any vertex of odd degree, because every time we get to a vertex we must be able to leave it using a different edge. But in the graph of the bridges, all the vertices are touched by an odd number of edges.





Problems:

a. the justification for believing that things stand in that particular way comes after and as a consequence of the mathematical reasoning. How do we know that the bridges in Königsberg cannot be crossed in that particular way? The mathematical facts in these example are not used to answer a “why is X the case?” type of question, but a “what reason do we have for believing that X is the case?” question. What Euler did was not to give an explanation but to provide a powerful justification for believing that it is impossible to find a route that would allow one to cross all the bridges only once. That this is so was not known independently before Euler’s treatment of the problem – it was at most suspected.

Problem: why are there so many philosophers that tend to think that mathematics can and does play an explanatory role in science, in cases such as these? There are two opposite directions one can take in trying to answer this question: either argue that these philosophers have wrong intuitions about the role of mathematics in science, or argue that their intuition is good but that, for some reason, they draw the wrong conclusions from it.

What relation is there between explanation and understanding?

Peter Achinstein: there is a "fundamental relationship between explanation and understanding" (Achinstein 1983, p. 16).

Wesley Salmon: "understanding results from our ability to fashion scientific explanations" (Salmon 1984, p. 259).

Michael Friedman takes the task of a theory of explanation as being that of telling us "what it is about the explanation relation that gives us understanding of the explained phenomenon, that makes the world more intelligible" (Friedman 1974, p. 7).

James Woodward's: a theory of explanation "should give us some insight into how such explanations work in the sense that it identifies the features or structures in virtue of which they convey understanding" (Woodward 2003, p. 23).

Michael Strevens, more recently, contends that scientific understanding of a phenomenon is just the state produced by grasping the correct scientific explanation of that phenomenon (Strevens 2008, p. 3).

What relation is there between explanation and understanding?

A new perspective

- de Regt, Grimm, Kvanvig, Elgin, Pritchard and many others, argue recently that understanding is (separately) worthy of philosophical attention and that its study can contribute new insights to many traditional debates in philosophy.

The possibility of understanding without explanation

- Kvanvig (2009), Lipton (2009), Gijsbers (2013) and Hindriks (2013)

My aim: to argue that we have strong reasons to be suspicious about any view that amounts to something like this: “there is no route to scientific understanding that does not go by way of scientific explanation” (Strevens 2013, p. 510). And to propose a new view on understanding, one that will help with our main task, i.e. that of showing that in the cases discussed the mathematical part is important for understanding something about the physical phenomena.

Is understanding a phenomenon the same thing as *having* a correct explanation of that phenomenon?

Case 1: suppose someone who has no knowledge of physics receives an explanation in terms of the law of conservation of angular momentum for why a rotating figure skater gains more speed just by drawing her arms closer to her body. Does she understand why this is happening?

- if the objectivist is right, the answer must be yes, but there is a strong intuition preventing us to accept such an answer.
- possible solution: the person in this example has a modest amount of understanding or none at all because she lacks a more detailed knowledge of the information contained in the explanation (Khalifa p. 21). Knowing an explanation presupposes a lot more than just hearing or reading one given in outlandish terms, it implies "having a mostly true and highly virtuous set of beliefs about the information constituting an explanation" (Khalifa 2012, p. 18).

Is understanding a phenomenon the same thing as *knowing* a correct explanation of that phenomenon?

Case 2: in an ideally large random mating and unaffected by mutation factors diploid biallelic population, reproduction will not affect genotypic variability (i.e. allele frequencies will not change from one generation to the next). Why is that?

Let's take p and q to stand for the frequency of allele A, respectively allele a, and P_{AA}, P_{Aa}, P_{aa} for the frequencies of the associated genotypes. We can calculate p and q this way:

$$p = \frac{1}{2}P_{Aa} + P_{AA}$$

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$$\begin{aligned} P'_{AA} &= P^2{}_{AA} + P_{AA}P_{Aa} + \frac{1}{4}P^2{}_{Aa} \\ &= (P_{AA} + \frac{1}{2}P_{Aa})^2 \\ &= p^2 \end{aligned}$$

$$\begin{aligned} P'_{Aa} &= P_{AA}P_{Aa} + 2P_{AA}P_{aa} + P_{Aa}P_{aa} + \frac{1}{2}P^2{}_{Aa} \\ &= 2(P_{AA} + \frac{1}{2}P_{Aa})(P_{aa} + \frac{1}{2}P_{Aa}) \\ &= 2pq \end{aligned}$$

P'... and ***p'***... stand for the frequencies in the next generation.

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$$P'_{aa} = \frac{1}{4}P^2{}_{Aa} + P_{Aa}P_{aa} + P^2{}_{aa} = (P_{aa} + \frac{1}{2}P_{Aa})^2 = q^2$$

Knowing this, it is easy now to calculate the allele frequencies for the next generation. This is how it's done:

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$$p' = \frac{1}{2}P'_{Aa} + P'_{AA} = p^2 + pq = p(p+q) = p$$

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Case 2…

Suppose now that this explanation is known by a biologist with a decent knowledge of elementary mathematics, but that this biologist has big problems in following any type of mathematical reasoning. Can we say that this person has understanding of the Hardy-Weinberg law?

- There is nothing more to this explanation than the mathematical reasoning, so, if one cannot follow it, how can she understand anything about this law?

Is understanding a phenomenon just a matter of *grasping* a correct explanation of that phenomenon?

If understanding is not equivalent with having/knowing an explanation it doesn't mean it is not nonetheless inextricably linked with it. Someone can accept what we have said earlier about the biologist and yet argue that it is impossible to have unexplanatory understanding.

"*An individual has scientific understanding of a phenomenon just in case they grasp a correct scientific explanation of that phenomenon*" (Strevens 2013).

In Strevens's view, grasping is more than just knowing that something is the case. Grasping is a kind of understanding: it is understanding that or direct apprehension, which resembles a sort of conscious realization that something is the case. For example, grasping that the cat is on the mat means being fully aware of the cat, the mat, and the spatial relation between them (Strevens 2013, p. 511).

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Case 2 reconsidered: If we look from this new perspective at the example discussed earlier, we can say this: due to her inability to follow the mathematical reasoning, the biologist, even though she has all the relevant knowledge of the explanatory information, cannot grasp the explanation and so cannot understand why the Hardy-Weinberg law is true.

Problems with the strong link view: it needs to rule out the possibility that understanding can be obtained in some other way than just by grasping an explanation. Once we disentangle understanding from explanation, this possibility can no longer be ignored. In order to rule out unexplanatory understanding, two things need to be shown: (**a**) that understanding cannot be obtained by grasping other things than explanations, and (**b**) that understanding always involves grasping.

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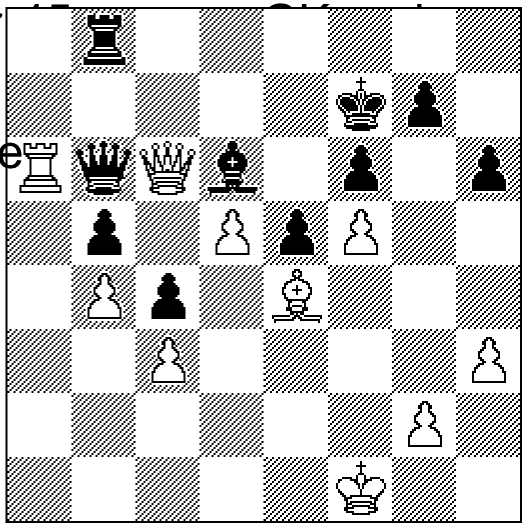
- Lipton takes understanding to be the cognitive benefit provided by explanations and argue that it can also be obtained "by routes that do not pass through explanation" (Lipton 2009, p. 44), e.g. by the use of images and physical models, by manipulation or by internalisation of Kuhnian exemplars.
- Gijsbers (2013) argues that that there is a type of unexplanatory understanding that we can acquire through correct classification of phenomena. In his view, "when we realise which phenomena are like which other phenomena, which phenomena ought to be unified under the same concept and which ought to be kept separate, we advance our understanding" (Gijsbers 2013, p. 521).

- both of them take understanding not as grasping something but as a form of knowledge (for Gijsbers it is knowledge of connections between phenomena while for Lipton it is knowledge of causes, of necessity, of possibility and of unification)

Is understanding a phenomenon just a matter of *grasping* a correct explanation of that phenomenon?

Case A: Two friends, Jane and Hilary, are watching together a game of chess between DB(white) and GK(black). Jane is a professional chess player but Hilary is only a novice. After



with the board looking like this:

- Jane understands, but Hilary has no clue why that happened. What does Jane have that Hilary lacks?

(i) explanation: we are not dealing with a kind of information that can receive explicit representation as explanations require.

(ii) knowledge: knowing how a similar game ended is insufficient for understanding in this case because in chess the fact that in a particular game a player won by a certain sequence of moves doesn't guaranty that the game couldn't have had a different outcome.

Jane understands in virtue of grasping the complex link between this particular positioning of chess pieces and a particular end of the game

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Case B: visual proof by Benjamin Klein and Irl Bivens (Nelsen 1993, p. 120) of the geometric series

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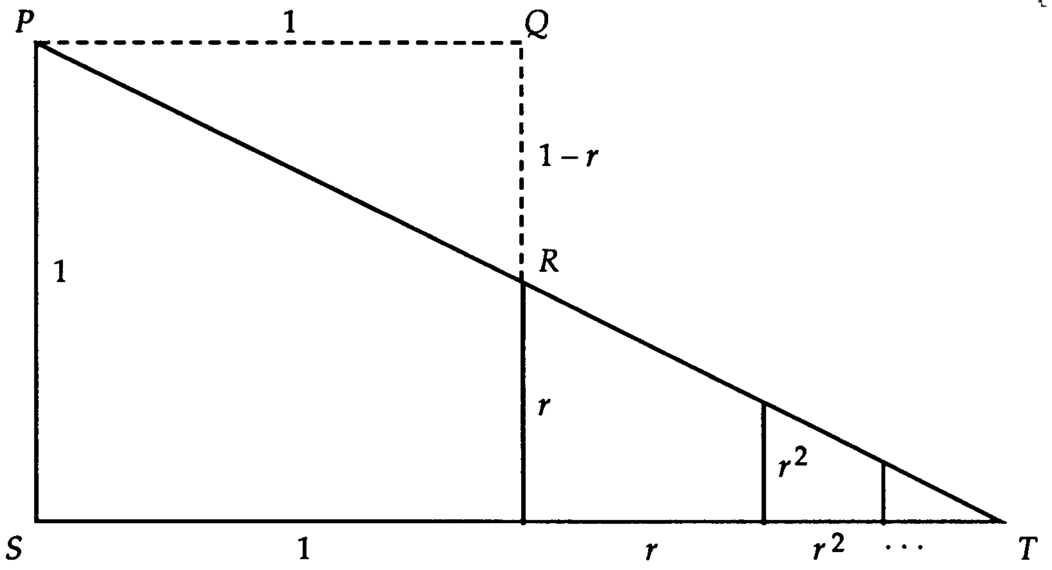
$$1 + r + r^2 + \cdots = \frac{1}{1-r}, \ (0 < r < 1)$$



The fact that we understand something has to do only with grasping the geometric relationships that hold in the figure below, most importantly that the triangles PQR and TSP are similar and so that the ratios of their corresponding sides must be the same.

Understanding is a cognitive achievement obtained by correctly exercising the ability to grasp relations of any kind: structural, explanatory, logical etc.

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Steiner's account: a characteristically mathematical explanation of a physical phenomenon is one in which, if "we remove the physics, we remain with a mathematical explanation – of a mathematical truth" (Steiner 1978, p. 19). The main idea here, even though not explicitly stated by Steiner, seems to be that explanatory power somehow leaks from the purely mathematical explanation into the mathematical explanation of the physical phenomenon.

Problems:

(1) In some of these examples, there is a proof for the mathematical theorems used, but this proof is not explanatory; so, there is no way that these explanations can inherit something from the pure mathematical counterpart (Baker 2012).

(2) In other examples we don't even have the luxury of debating if the proof of the mathematical result used is explanatory or not because there is no such proof to begin with (Baker 2012).

Is it possible to transfer explanations from one domain to another?

“Maintaining that there are intra-mathematical explanations but that these explanations never permeate beyond the boundaries of mathematics is prima facie implausible” (Baker and Colyvan 2011, p. 327).

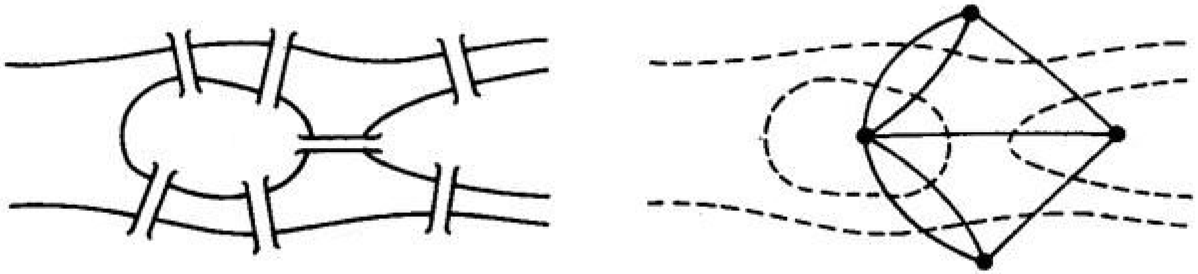
Against plausibility: things happen in a certain system only because of the facts that have to do with that system or its relations with other systems. But, if this is indeed the case, it means that any explanation of something happening in a certain system has to be given in terms of facts about that system, i.e. of the laws governing it, of the relations holding between its parts etc.

Against transfer: Suppose we have two noninteracting but analogue systems **A** and **B**, i.e. between them there can be a mapping relation based on similarity of relations. Suppose further that in **A** we have an explanation that tells us that some fact ***x*** is the case because of some other fact ***y***; but in **B** we have no such explanation linking x^B with some y^B. Now, according to Steiner, given the fact that there is a mapping relation between **A** and **B**, we can use whatever explanation we have in **A** to account for x^B’s occurrence in **B** *in terms of* ***y***. But this make no sense. The most that the existence of such a relation between **A** and **B** allows us to do is to infer that there is a y^B in **B** that may have something to do with x^B being the case. Depending on the type of relation between **A** and **B**, our *knowledge why* things are in a certain way in **A** can only be transformed into a *belief that* things may also be in a similar way in **B**, or, if we are dealing with an isomorphism, into *knowledge that* things must be the same in **B**.

Is it possible to transfer understanding from one domain to another?

In my view, understanding is a cognitive achievement obtained by correctly exercising the ability to grasp relations of any kind: structural, explanatory, logical etc

Let's reconsider the case discussed earlier: we have two noninteracting but analogue systems **A** and **B**, i.e. between them there can be a mapping relation based on similarity of relations. Suppose further that in **A** we have understanding (obtained in no matter what way) of the relation ***R*** holding between some fact ***x*** and some other fact ***y***; but in **B** (for whatever reason) we have no such understanding. Can we transfer the understanding we have in **A** to **B**? Yes, we can! Unlike the case of explanation, here we are not dealing with a change of subject when we change systems. $\boldsymbol{R_{xy}}$ is the same thing as $\boldsymbol{R_{xByB}}$ so, if we understand something about one we automatically understand the same thing about the other.



The end.

Thank you!