

# Ibn al-Haytham's Geometrical Methods and the Philosophy of Mathematics A History of Arabic Sciences and Mathematics Volume 5

**Edited by <u>Roshdi Rashed</u>** © 2017 – Routledge 674 pages, avril 2017

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This fifth volume of *A History of Arabic Sciences and Mathematics* is complemented by four preceding volumes which focused on the main chapters of classical mathematics: infinitesimal geometry, theory of conics and its applications, spherical geometry, mathematical astronomy, etc.

This book includes seven main works of Ibn al-Haytham (Alhazen) and of two of his predecessors, Thābit ibn Qurra and al-Sijzī:

- The circle, its transformations and its properties;
- Analysis and synthesis: the founding of analytical art;
- A new mathematical discipline: *the Knowns*;
- The geometrisation of place;
- Analysis and synthesis: examples of the geometry of triangles;
- Axiomatic method and invention: Thabit ibn Qurra;
- The idea of an Ars Inveniendi: al-Sijzī.

Including extensive commentary from one of the world's foremost authorities on the subject, this fundamental text is essential reading for historians and mathematicians at the most advanced levels of research.

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