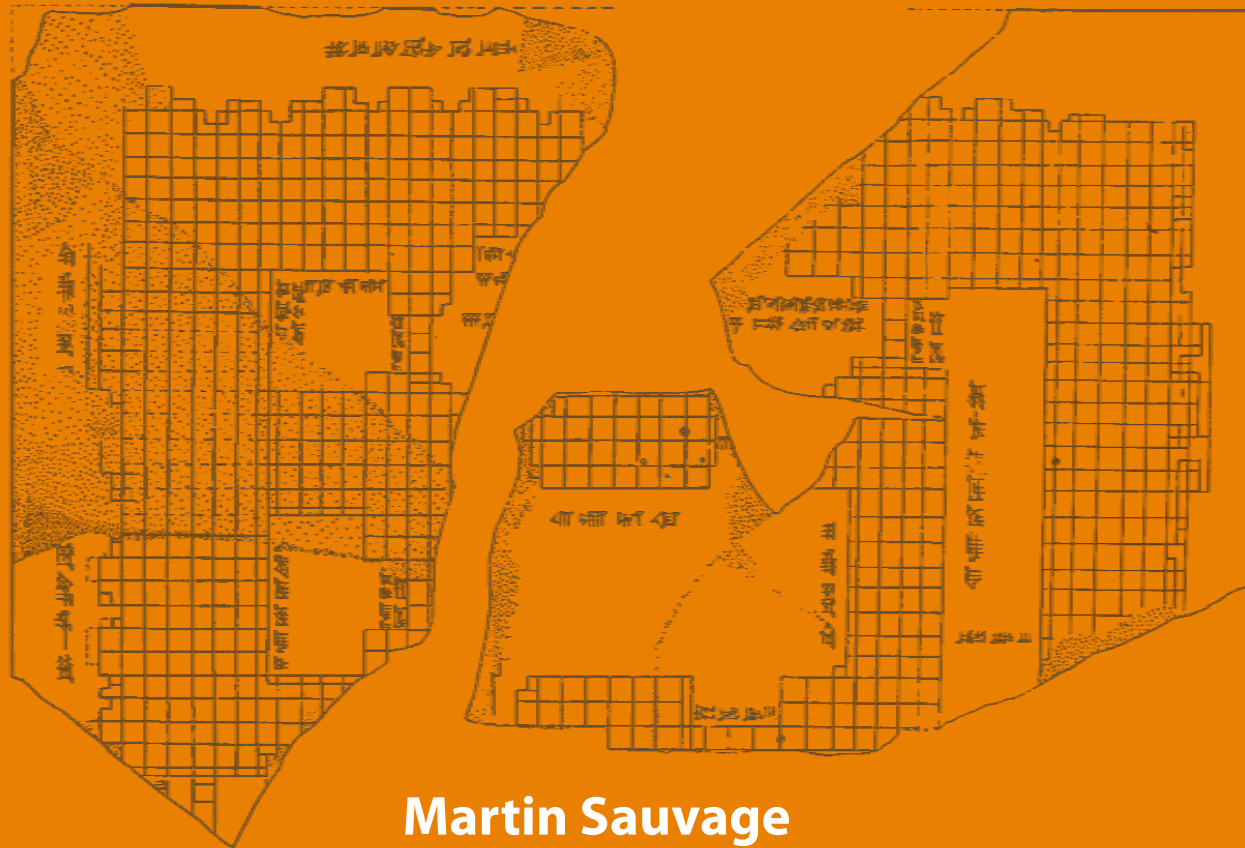


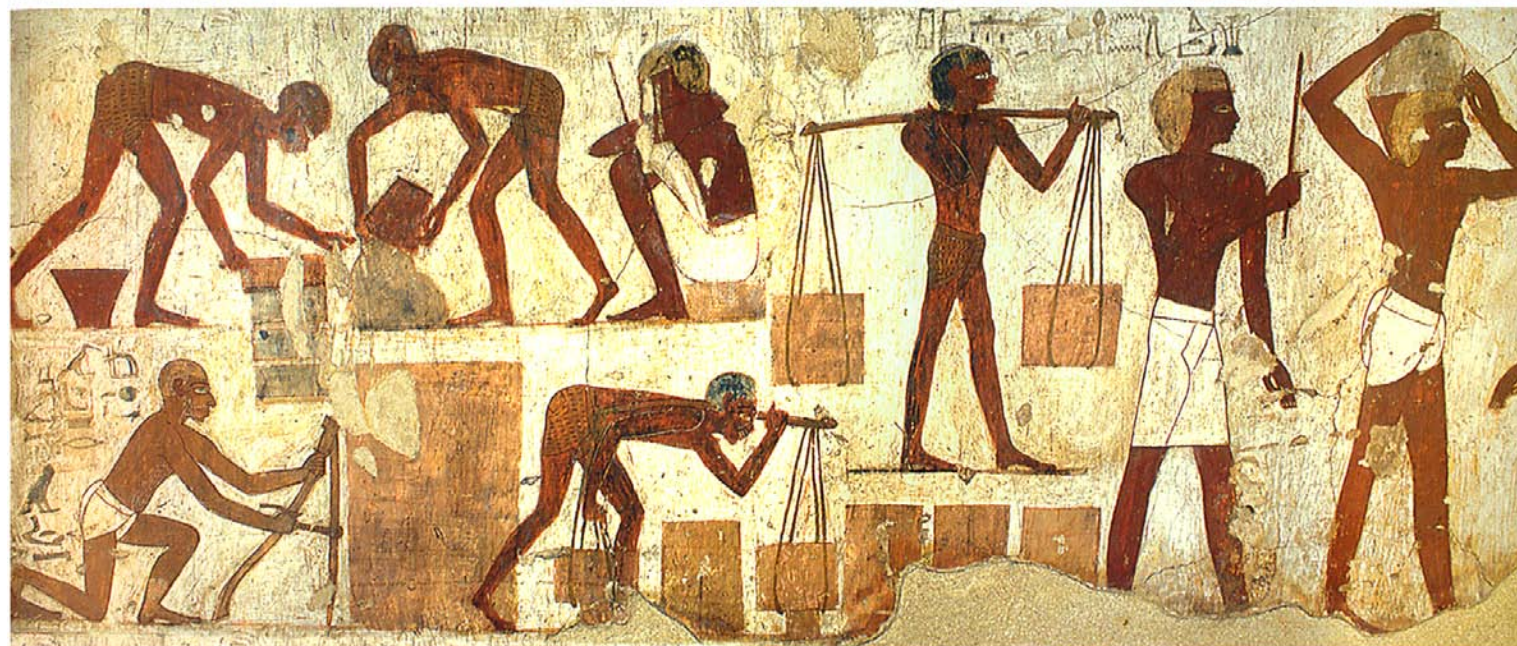
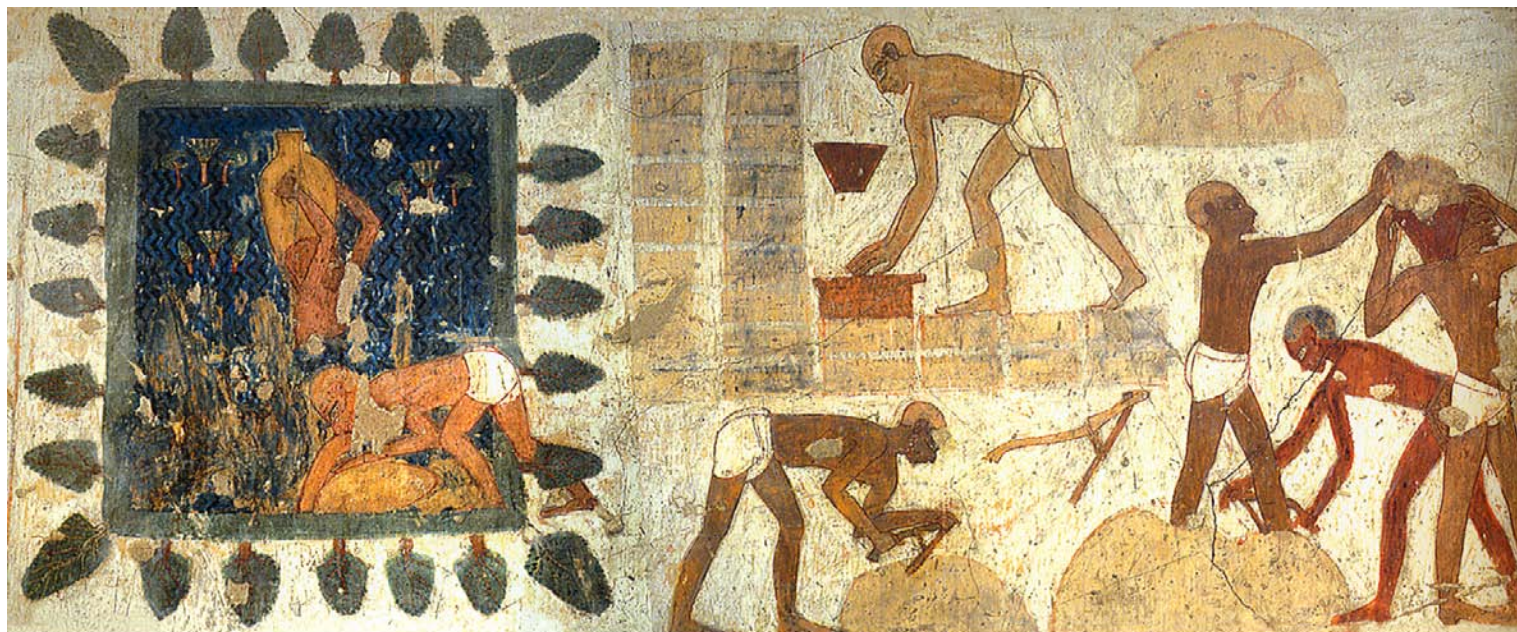
*Mathematical computation
in the management of public works in Mesopotamia
(End of the 3rd and beginning of the 2nd millennium BC).*

SAW seminar, 6 April 2012 - **Constructions and excavations**



Martin Sauvage

CNRS, USR 3225, Société préhistorique française
et UMR 7041, équipe « Histoire et ARchéologie de l'Orient Cunéiforme »



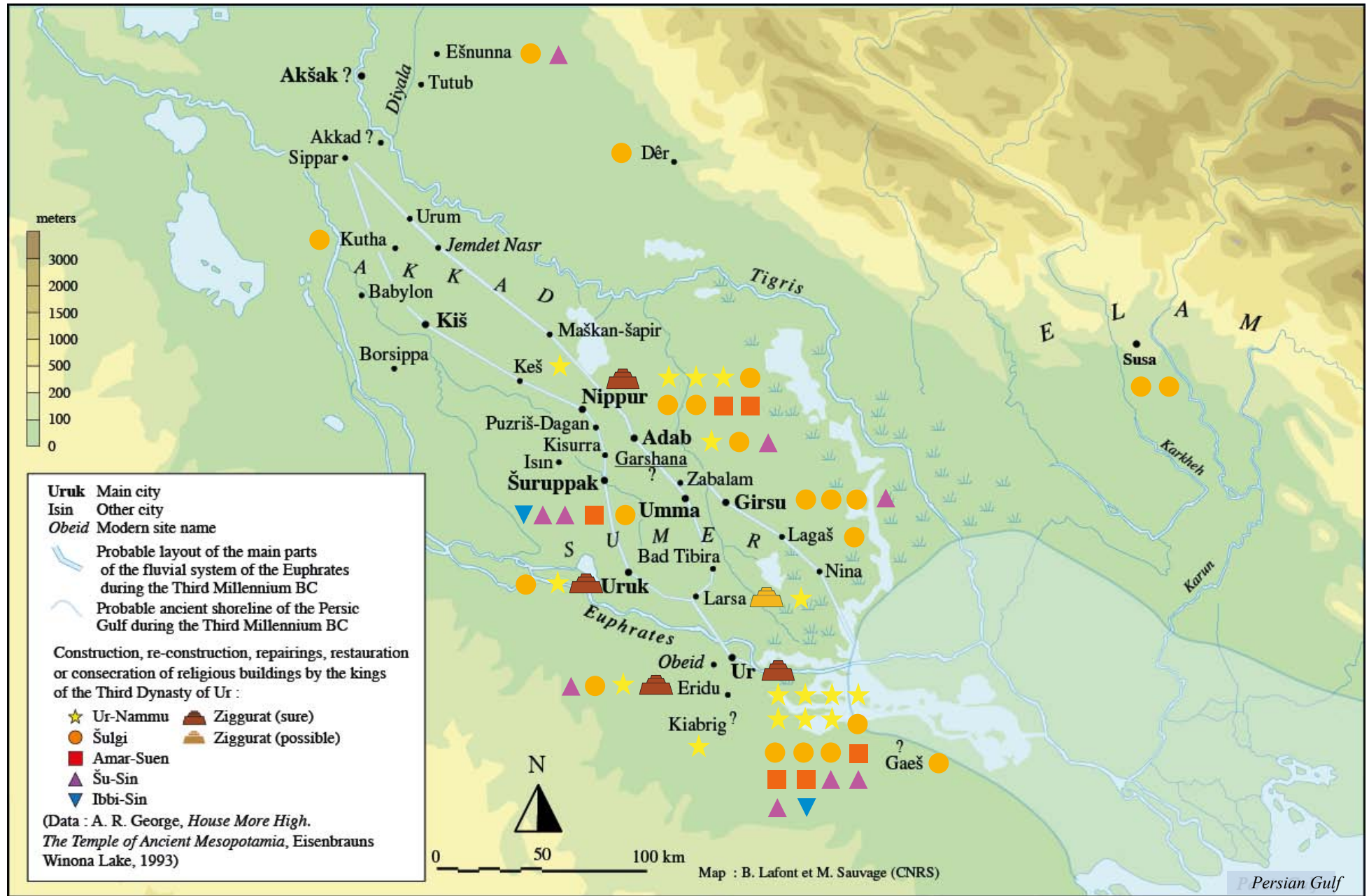
Tomb of Rekhmirê, Thebes (Egypt, XVIIIth dynasty)



Mixing clay with water and straw



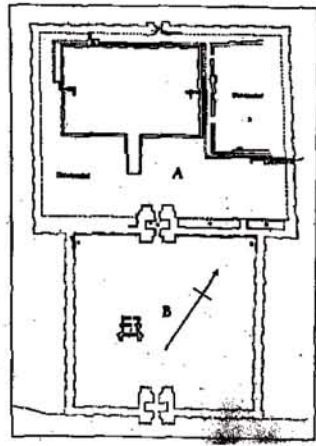
Brick moulding



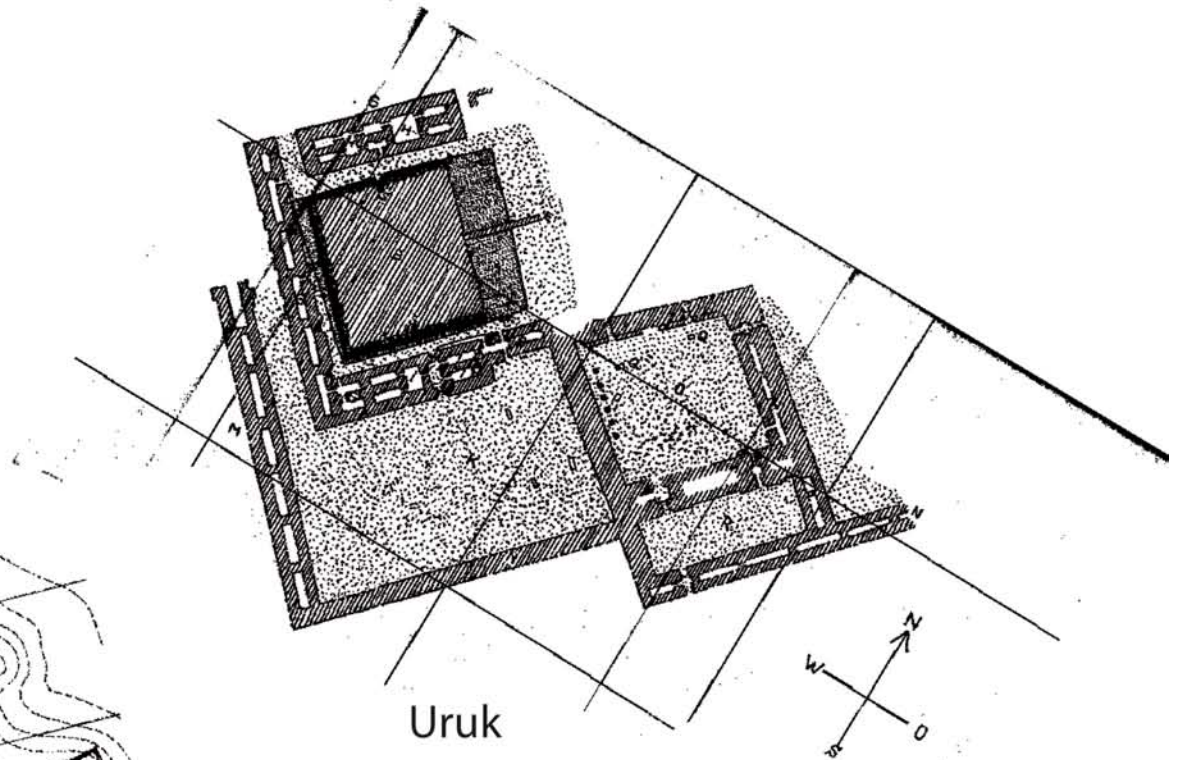
Construction of religious buildings during the Third Dynasty of Ur



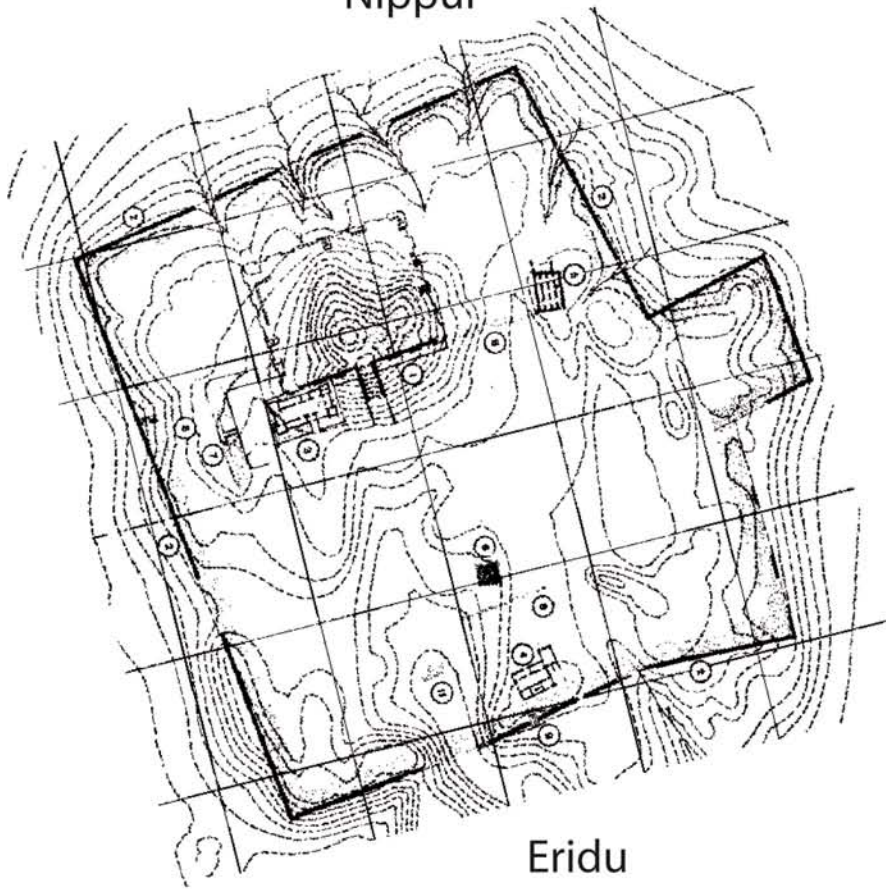
The Ziggurat of Ur after excavations and before restoration



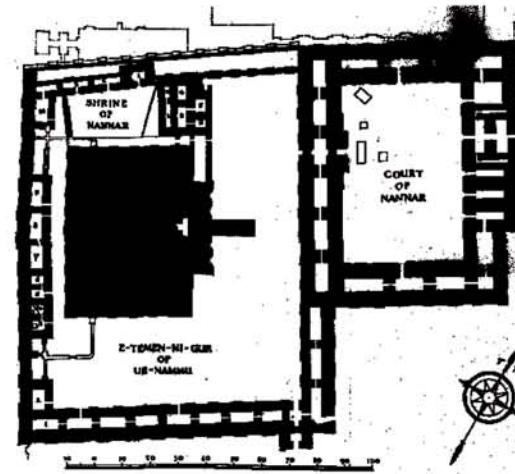
Nippur



Uruk



Eridu



Ur

The four Ur III ziggurats

SUMERO-BABYLONIAN METROLOGICAL UNITS

Mass

- 1 load (gín / *biltu*) = 60 pounds \approx 30 kg
- 1 pound (ma-na / *manû*) = 60 shekels \approx 500 g
- 1 shekel (gín / *šiq̄lu*) = 180 grains \approx 8.33 g
- 1 grain (še / *uṭṭetu*) \approx 0.046 g

Distance

- 1 league (dana / *bêru*) = 180 cords \approx 10.8 km
- 1 cord (éše / *ašlu*) = 10 rods \approx 60 m
- 1 rod (ninda / *nindânu*) = 12 cubits \approx 6 m
- 1 reed (gi / *qanû*) = 6 cubit \approx 3 m
- 1 cubit (kùš / *ammatu*) = 30 fingers \approx 50 cm
- 1 finger (šu-si / *ubânu*) = 6 grains \approx 1.66 cm
- 1 grain (še / *uṭṭetu*) \approx 0.28 cm

Surface

- 1 bùr (bur / *buru*) = 18 iku \approx 6.48 ha
- 1 iku (iku / *kû*) = 100 sar \approx 0.36 ha
- 1 sar (sar / *mušaru*) \approx 36 m² (0.0036 ha)

Volume

- 1 iku (iku / *kû*) = 100 sar \approx 1,800 m³
- 1 sar (sar / *mušaru*) \approx 18 m³
- 1 sar = 5,00,00 [= 18,000] sila (\approx 180 hl)

Capacity

- 1 gur (gur / *kurru*) = 5 bariga \approx 300 l
- 1 bariga (bariga / *pânu*) = 6 bán \approx 60 l
- 1 bán (bán / *sûtu*) = 10 silà \approx 10 l
- 1 silà (silà / *qû*) = 60 shekels \approx 1 l
- 1 shekel (gín / *šiq̄lu*) \approx 16.66 ml

Bricks

- 1 bùr = 1800 sar = 1,296,000 bricks
- 1 ešè = 600 sar = 432,000 bricks
- 1 iku = 100 sar = 72,000 bricks
- 1 sar = 60 gín = 720 bricks
- 1 gín = 12 bricks

- 1 sar_a = 1 ninda × 1 ninda (areas, surfaces)
- 1 sar_v = 1 ninda × 1 ninda × 1 cubit (volumes)
- 1 sar_b = 720 bricks (when counting bricks, whatever the brick type)

Tasks	Daily task	Daily task (m ³)	Reciprocal: number of days for a sar _v (or 1 ninda)
Stacking bricks	0;04,10 sar _v	1,13	14;24 (14,04) days
Building a dam?	0;05,20 sar _v	1,53	11;15 (11,25)
Building an earthen wall	0;03,45 sar _v	1,125	16
Building a dike	0;10 sar _v	3	6
Cutting reeds	0;10 sar _v	3	6
Digging (less than a cubit)	0;20 sar _v	6	3
Digging (more than a cubit)	0;10 sar _v	3	6
Mixing earth	0;10 sar _v	3	6
Destroying a wall	0;15 sar _v ; 0;20 sar _v	4,5	4 or 3
Moulding bricks (sundried bricks?)	0;25 sar _v	7,5 m ³ (2161 type 2 bricks, 809 type 8)	2;24 (2,4)
Moulding bricks (baked bricks?)	0;20 sar _v	6 m ³ (1729 type 2 bricks, 647 type 8)	3

Iškarum: daily tasks for workers

□ AO 5676: VI 11-12 [Ur III adm. text, Umma: Genouillac 1922: n° 5676; Maekawa 1990; **Robson 1999: 139**)

Cutting reeds.

4 05 sar gi ku₅-rá 10 sar-ta
á-bi u₄ 24 ½

4,05 sar_a of cutting reeds at 10 sar_a (a day):
its work is 24 ½ days.

8820 m² at 360 m² a day, that means 24.5 man-days

◆ Cutting reeds: 10 sar_a/day (360 m²/day)

*

□ AO 5676 [Ur III adm. text, Umma. Genouillac 1922: n° 5676; Maekawa 1990; Robson 1999, p. 139, 152]

Lines VI 11-12

4 05 sar gi ku₅-rá 10 sar-ta
á-bi u₄ 24 1/2

4,05 sar_a of cutting reeds at 10 sar_a (a day)
its work is 24 ½ days.

Lines IX 13-14

33 guruš u₄-1šè
gí ÍL-gá

33 workers for a day
carrings reeds

4,05 sar_a (8,820 m²) with 360 m² /day = 24.5 man-days.

◆ Cutting reeds: 10 sar_a/day (360 m²/day)

□ *DV 5 64* [Akkad adm. text, unknown provenance. Nikolskii 1915: n° 64; Powell 1976a; **Robson 1999: 139**]

Dimensions of 3 canal banks, total volume: 14;30 sar_v (261 m³). Work duration: 87 days, thus 14;30/87 = 0;10 sar_v/day (Powell 1976b: 100-102; Friberg 1982: 117-118). This is the usual value in OB mathematical texts.

◆ Excavation work: 0;10 sar_v/day (3 m³/day)

*

□ *MLC 2404 1-4* [Ur III adm. text, Umma. Keiser 1914 : n° 116; **Robson 1999: 140**]

Earthworks

35 1/3 sar kin-saḥar

á lú-ḥun-gá

[...] sīla-ta

á-bi u₄-3 32-kam guruš u₄-1-šè

35 1/3 sar_v earthworks.

The wages of the hired men

are ... litres each:

its work is 3,32 labourers for a day

636 m³ of earth is 3,32 (212) man-days. [636/212 = 3 m³/day]

◆ Earthwork: 0;10 sar_v/day (3 m³/day)

*

□ *YBC 3668: IV 64-66* [Ur III adm. text, Umma. Keiser 1919: n° 209; **Robson 1999: 140**]

Repairs to a dike

3 40 sar saḥar

á lú-ḥun-gá 6 sīla-ta

še-bi 26 (gur) 2 (bariga)

3,40 sar_v earth.

The wages of the hired men are 6 litres each:

its grain is 26 gur, 2 bariga

66 m³ earth; total of the wages 7920 l. 7920/6 = 1320 days, or 3 m³ /day

◆ Earthwork: 0;10 sar_v/day (3 m³/day) and wages: 6 l of barley/day.

Carrying materials	volume of a load (sar _v)	Daily vol. per ninda (sar _v)	Daily vol. per ninda (m ³)	Daily vol. for 30 ninda = 180 m (m ³)	Equivalent in earth baskets
Earth	0;00,02,13,20	1;40	29,88	1	1
Bricks	0;00,04,10	3;07,30	56,25	1,875	1 7/8
Straw	0;00,04,26,40	3;20	59,94	2	2
Reeds	0;00,06,40	5;00	90	3	3
Water	0;00,01,46,40	1;20	23,94 (= 24.000 l)	0,7798 (800 l)	4/5

$\frac{\text{total volume or surface}}{\text{daily task}} \rightarrow \text{numbers of days of work}$

$\frac{\text{earth carrying coefficient}}{\text{distance of each trip}} \rightarrow \text{volume of earth carried in a day}$

$\frac{\text{going}(ninda / day) \times \text{basket}(sar_v)}{\text{distance}(ninda)} \rightarrow \text{daily rate of carrying}(sar_v / day)$

Carrying building materials:

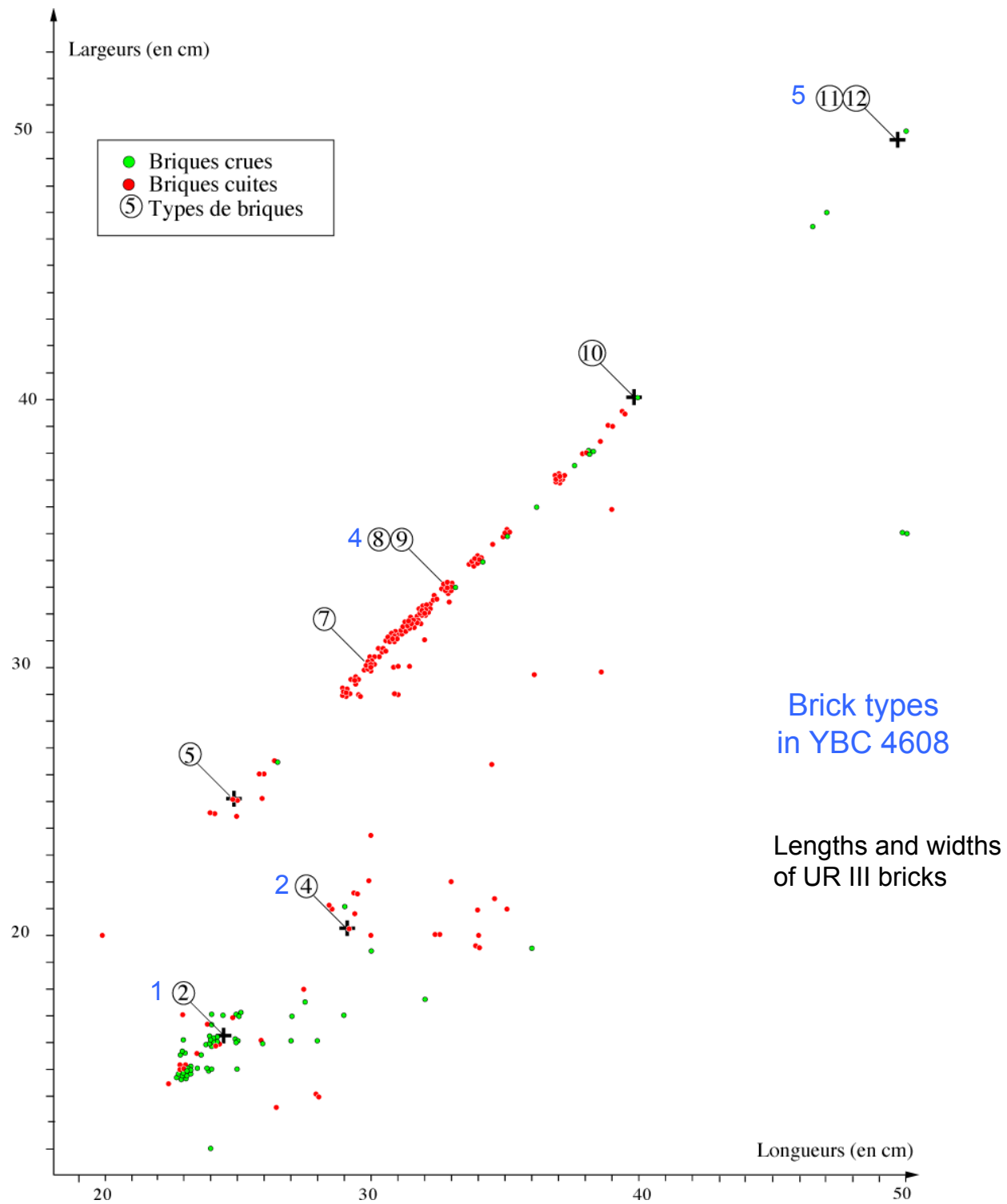
tupšikkum (the “basket”) = the worker load

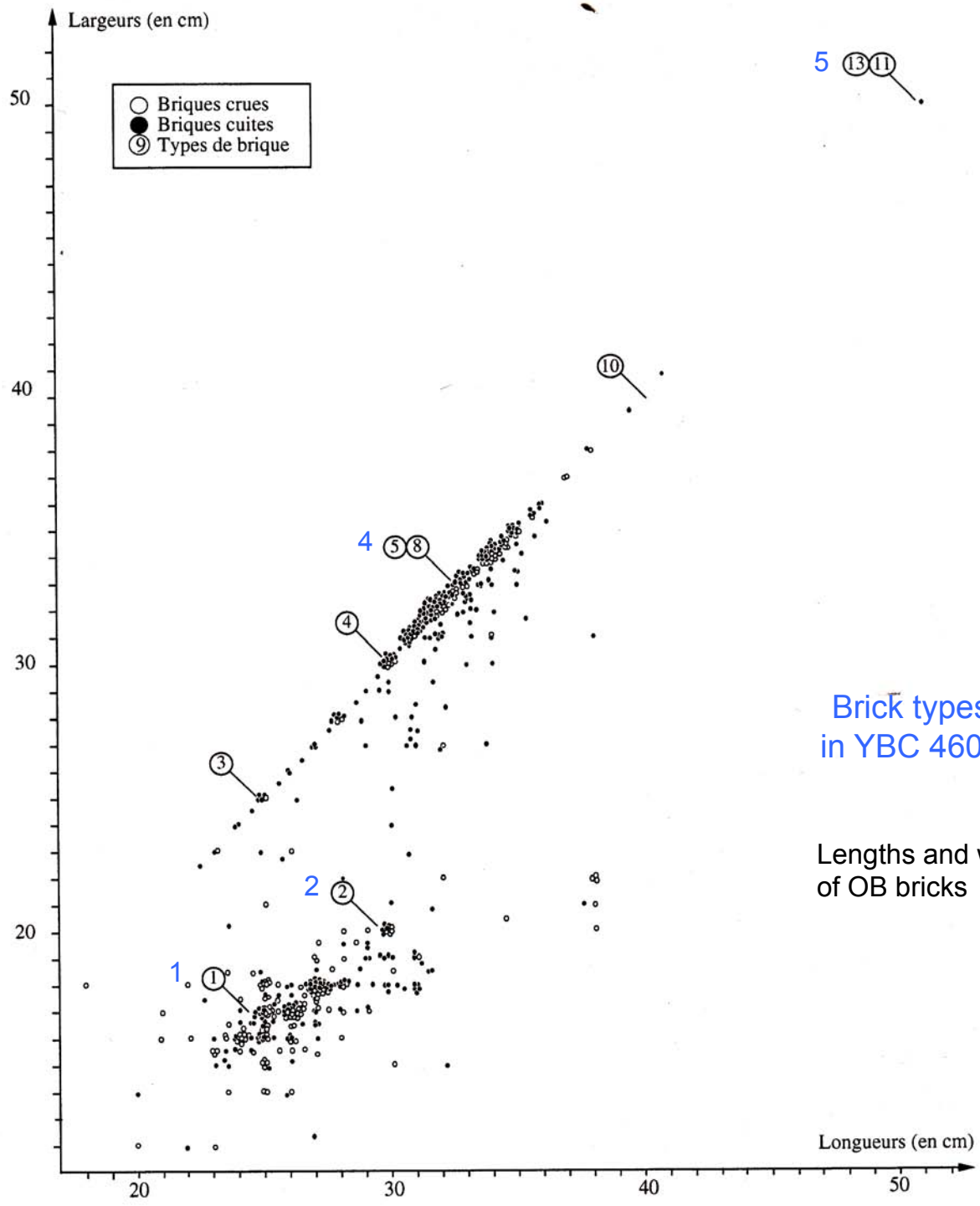
muttalliktum (the “going”) = the daily distance

Brick Types	dim. (fingers)	dim. (cubits)	dim. (cm)	Notes	
1	$10 \times 10 \times 6$	$1/3 \times 1/3 \times 1/10$	$17 \times 17 \times 10$	Quarter of 9	
1a	$12 \times 9 \times 6$	$1/5 \times 3/10 \times 1/10$	$20 \times 15 \times 10$		
2	1	$15 \times 10 \times 5$	$1/2 \times 1/3 \times 1/12$	$25 \times 17 \times 8$	
3	3	$20 \times 10 \times 5$	$2/3 \times 1/3 \times 1/12$	$33 \times 17 \times 8$	Half of 8
4	2	$18 \times 12 \times 5$	$3/5 \times 2/5 \times 1/12$	$30 \times 20 \times 8$	
5		$15 \times 15 \times 5$	$1/2 \times 1/2 \times 1/12$	$25 \times 25 \times 8$	
6		$20 \times 10 \times 6$	$2/3 \times 1/3 \times 1/10$	$33 \times 17 \times 10$	Half of 9
7		$18 \times 18 \times 5$	$3/5 \times 3/5 \times 1/12$	$30 \times 30 \times 8$	
8	4	$20 \times 20 \times 5$	$2/3 \times 2/3 \times 1/12$	$33 \times 33 \times 8$	
8a		$30 \times 15 \times 5$	$1 \times 1/2 \times 1/12$	$50 \times 25 \times 8$	Half of 11
9		$20 \times 20 \times 6$	$2/3 \times 2/3 \times 1/10$	$33 \times 33 \times 10$	
10		$24 \times 24 \times 5$	$4/5 \times 4/5 \times 1/12$	$40 \times 40 \times 8$	
11	5	$30 \times 30 \times 5$	$1 \times 1 \times 1/12$	$50 \times 50 \times 8$	
12		$30 \times 30 \times 6$	$1 \times 1 \times 1/10$	$50 \times 50 \times 10$	

Brick types
in YBC 4608

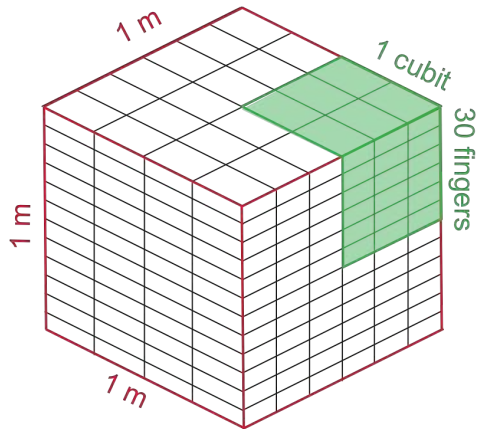
Brick types in OB mathematical texts



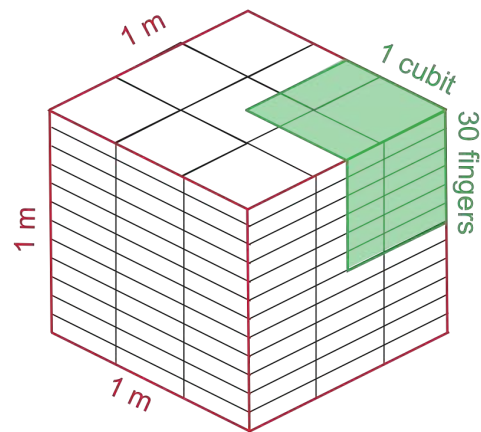


Brick types
in YBC 4608

Lengths and widths
of OB bricks



Type 2 sun-dried bricks (*sig₄ / libittum*)
 $1/2$ cubit (*kuš / ammatum*) \times $1/3$ cubit \times 5 fingers (*šu.si / ubânum*)
 (25 \times 16,66 \times 8,33 cm)
 36 bricks per cubit³
 5184 briques au sar_v
 24 bricks per m²
 288 bricks per m³



Type 8 baked bricks (*sig₄.al.ùr.ra / uggurum*)
 $2/3$ cubit \times $2/3$ cubit \times 5 fingers
 (33 \times 33 \times 8,33 cm)
 13,5 bricks per cubit³
 1944 briques au sar_v
 9 bricks per m²
 108 bricks per m³

TWO EXAMPLES OF OB MATHEMATICAL TEXTS

- ❑ YBC 4669 (xii) [Part of an OB mathematical text, unknown provenance. Robson 1999: 66].

Calculations on a brick-pile.

The length is $3\frac{1}{2}$ nindan, 3 cubits. The bricks are $\frac{1}{3}$ cubit square. The height of the pile is 4 cubits. The width of the pile is 2 cubits. What is the volume of the brick-pile and how many bricks are there? The volume of the pile is 2;15 sar. There are 20 sar and 3 sixties of bricks.

*

- ❑ YBC 4673 (v) [Part of an OB mathematical text, unknown provenance. Robson 1999: 90].

Calculations on brick-making and construction of a brick-pile.

One man carried earth over a distance of 30 ninda and he built a brick-pile. For what proportion of the day he carry earth; for what proportion of the day did he built the brick-pile; and how many bricks were there? 2,40 [i.e. total of bricks: 160].

TWO EXAMPLES OF OB MATHEMATICAL TEXTS

- ❑ YBC 4669 (vi) [Part of an OB mathematical text, unknown provenance. Robson 1999: 89].

Combined coefficient exercise.

Let a man carry (earth) for a length of 20 nindan and let him demolish (a wall). For what proportion of the day shall he demolish (the wall), and for what proportion [of the day] shall he carry (the earth)? In a fifth of the day, or 0;12, he demolished 4 shekels (of earth); in 0;48, the remainder of the day, he carried 4 shekels (of earth).

*

- ❑ YBC 4673 (v) [Part of an OB mathematical text, unknown provenance. Robson 1999: 90].

Calculations on brick-making and construction of a brick-pile.

One man carried earth over a distance of 30 nindan and he built a brick-pile. For what proportion of the day he carry earth; for what proportion of the day did he built the brick-pile; and how many bricks were there? 2,40 [i.e. total of bricks: 160].

List of tasks for the construction of a storehouse [é-kišib-ba: Robson], or a school [é-dub-ba: Farber].

Transcription (Farber, 1989)	Robson 1999: 153
Vs 8 sar sig ₄ du ₈ -a	24 men to make 8 sar of bricks [5,760 bricks]
. a ₂ -bi 24 ġuruš u ₄ -1-še ₃	16 men to carry 8 bricks each over 60 nindan
16 ġuruš u ₄ -1-še ₃	8 men to mix mortar
sig ₄ 8-ta IL ₂ -ġa ₂	9 men to carry 90 reed-bundles for mats
5 kaskal-bi 60 ninda UŠ	3 men to mix mortar
a ₂ al-tar-a gub-ba-bi 8 ġuruš u ₄ -1-še ₃	18 men to carry 180 reed-bundles for putting into the brickwork
90 sa-gi ti-um-ma-bi	6 men to mix mortar
a ₂ ġuruš gi IL ₂ -bi 9 ġuruš u ₄ -1-še ₃	Total : 84 man-days
a ₂ ġuruš al-tar-a-bi 3 ġuruš u ₄ -1-še ₃	
10 180 sa-gi	
Rs e ₂ -a anše si-ga u ₃ ħu-ur ₂ -da keš ₂ -DU	
. a ₂ gi IL ₂ -bi 18 ġuruš u ₄ -1-še ₃	
a ₂ ġuruš al-tar-bi 6 ġuruš u ₄ -1-še ₃	
šu-nigin ₂ 84 a ₂ ġuruš u ₄ -1-še ₃	
15 e ₂ -dub-ba' [or : kišib-ba ?] ša ₃ e ₂ -amar-ra-ka du ₃ -a	
ki šu- ^d Adad-ta	
ba-zi	
mu Ki-maš ^{ki} ba-ħul	

240 bricks a day for moulding do not correspond to OB mathematical coefficients: one gives 0;25 sar_v/day or 2,161 bricks/day of Type 2 bricks; one other gives 0;20 sar_v/day or 1,729 bricks/day of type 2 bricks).

The task probably consists here in both carrying and mixing earth, and moulding bricks. But in this case, we do not know the distance for carrying earth before mixing, or carrying the mixture before moulding. An average value of 240 bricks/day for brick making is often attested in both Ur III (e.g. Garshana, see Heimpel, 2009: 224) or OB administrative texts (e.g. NBC 5506, see Robson 1999: 153).

98-07-66 Brick carrier text**I.**

1. [?] sig₄-hi-tum
 2. [n] a-hu-wa-qar
 3. [n] im-ti-dam
 4. [n] nu-ri-li₂
 5. [n] um-mi-da-hi
 6. [n] ba-ku-tum
 7. [n] la-ma-za-t[um]
 8. [n] za-zi-x-tum
 9. [n]
 10. [n] x-ki-um-mi
 11. [n] na-a-a?
 12. 720 ša-IGI.DU-na?
 13. 450 zi-ip-ri-tum
 14. 720 ša-at-i₃-li₂
 15. 240 [i]m?-ta-tum ugula? /
 16. 240 x[. . . .]
 17. 7800[+n]
 18. nam-10 š[a-at-. . . .]
 19. 600 en-um-am?-na?-a?
 20. 600 ša-at-er₃-ra
 21. 600 im-ti-dam
 22. 240 a-zi-num₂
 23. 240 d.adad-nu-ri
 24. 240 ša-at-i₃-li₂
 25. 120 x-ra-x-at
 26. 120 a-ni-a-hi
- II.**
27. 360 du-ni-[. . . .]
 28. 240 x-x-tum
 29. 720 [t]a?-ti-la?
 30. [2520+]240
 31. [n] im-ti-dam
 32. [n] nu-ur₂-i₃-li₂

33. [n] a-da-lal₂
 34. 240 er₃-ra-an-dul₃
 35. 720 puzur₄-a-bi
 36. 480 im-ti-dam
 37. 840 a-hu-ni
 38. 840 šu-er₃-ra
 39. 840 u-bar-um
 40. 360 bi₂-x-um
 41. 1380 [. . . .]
 42. 480 [. . .g]i-ba-ša
 43. [n]-ga
 44. [n]-hi-i₃-li₂
 45. [n]-x-x
 46. [n+]24
 47. nam-10 [a?]-hu-ni
 48. 240-ti
 49. 300 ba-ra-x-ti
 50. 240 simat-d.nin-šubur
 51. 240 geme₂-d.nin-lil₂
 52. 240 ur-me-lam₂
 53. 240 d.adad-[. . . .]
 54. 240 d.šamaš-[a-bi]
- III.**
55. [n]
 56. 360 [. . . .]
 57. 300 [. . . .]
 58. 3000
 59. nam-10 d.adad-[. . . .]
 60. [n] na-ab-[. . . .]
 61. [n] e₂-a-[. . . .]
 62. [n] a-da-[lal₂]
 63. 600? a-ni-[. . . .]
 64. [n] mi-ni-šu-da
 65. 600 ša-at-i₃-li₂
 66. 600 ša-at-er₃-ra

67. [n] a-ni-a-hi
 68. [n] a-ni-a-bi
 69. [n] d.adad-[nu]-ri
 70. 600 na-ab-r[i-tu]m
 71. 840 ša-hu-na-[x]
 72. 480 a-ni-a-hi min
 73. 480 la-ma-za-tum
 74. 720 i-zi-na[-x?]
 75. 720 ša-hi-ti[-x?]
 76. 8920
 77. nam-10 a-ni-a-bi
 78. 120 na-na-a
 79. 600 d.šamaš-a-bi
 80. 120 ga-[. . . .]
 81. 480 x[. . . .]
- IV.**
82. [. . . .]
 83. [. . . .]
 84. [n+]240
 85. BLANK SPACE
 86. šu+nigin₂ 48[+n]
 87. uš-bi 90
 88. a₂-geme₂-1 120 sig₄-ta
 89. geme₂-bi 288 - u₄-1-še₃
 90. še-bi 2.4.2.4 sila₃ / gur
 91. [. . . .] sig₄ il₂-ga₂
 92. [. . . .] e₂-uš-bar-še₃
 93. [. . . .] simat-e₂-a ugula / lu₂-hun-ga₂
 94. [gi]ri₃ dad[ad-il]at šabra
 95. [u₃] puzur₄-d.[nin-k]ar-ak / du[b-sa]r
 96. [ša₃ gar-š]a-an-na.[ki]
 97. iti a₂-k]l-ti
 98. [mu d.šu-d.EN].ZU lugal / [. . . .]