Indian Journal of History of Science, 46.2 (2011) 251-311

PEACOCK'S *ARITHMETIC:* AN ATTEMPT TO RECONCILE EMPIRICISM TO UNIVERSALITY*

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(Received 4 May 2010; revised 8 February 2011)

When the Whig Anglican algebraist Rev. George Peacock (1791-1858) conceived of his new abstract view of Symbolical Algebra in the 1830s, he had already written an impressive little known (History of) *Arithmetic* for the *Encyclopaedia Metropolitana*, eventually published in 1845, back in the 1820s. This paper studies why his (History of) *Arithmetic* was conceived and how it reinforced Peacock's general view of algebra as a symbolizing process. As a fellow, tutor and lecturer at Trinity College since 1814, Peacock was involved very early in the renewal of mathematics curriculum and mathematical research in Cambridge, as well as in the cultivation and the diffusion of science. As a reformer, Peacock, along with his colleagues in Cambridge, faced the Industrial Revolution, its varied pressures on the country's academic institutions, and its concern with transformation processes. As soon as the 1820s, Peacock sought out a universal genesis from arithmetic to algebra, grounded on the mathematical language of operations, and he launched his (History of) *Arithmetic* as a large inquiry into the vocabulary that all known tribes and nations used for elementary computations. In this way, he supported a general empiricist approach to science, deeply rooted in Locke's philosophy of human understanding. With a comparative and philological approach to numeral languages in hand, Peacock presented first arithmetic and then algebra as the progressive developments of abstract calculating languages, symbolizing computational processes. This view accounted for the special place he gave to Indian and Arabic arithmetics in his exposition of contemporaneous knowledge on numbers.

Key words: Arithmetic, Algebra, Cambridge, History, Peacock, Symbolical

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* A first version of this paper was read at the XXIIth International Congress of History of Science (Beijing, 2005), both with Dhruv Raina and Agathe Keller first version papers.

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PRESENTATION: WHAT WAS AT STAKE ?

In 1826, the Whig Anglican algebraist Rev. George Peacock (1791-1858) wrote an impressive *History of Arithmetic* for the *Encyclopaedia Metropolitana*[1], which was only published in 1845. As historians of mathematics rather identify Peacock as one of the British algebraists who impulsed a new abstract way to conceive algebra in the first half of the 19th century, they generally ignore this encyclopaedic paper of 154 double-column pages. Since at least four decades, numerous historians of mathematics explored what was really brought at this turning point in algebra. For instance, Lubos Novy detailed each contribution of these British algebraists setting up the main realms of modern algebra[2], and Walter Cannon situated the first generation of them as the core of what he named « the network of Cambridge »[3]. More recently, historians of science explored further the contextual conditions of birth of this trend of thought.

The different stages of the contribution of this network for the renewal of algebra are now famous. Its first generation, formed by Charles Babbage (1791-1871), with John F.W. Herschel (1791-1871), Peacock and some today less known students, founded *The Analytical Society* in 1812, in order to enforce the introduction of the Leibnizian notation for the infinitesimal calculus in Cambridge[4]. They worked in publishing papers on this topic, and new text books for Cambridge examinations[5], in order to « reimport in England … a century of foreign improvement »[6], and to found a new view of algebra which could make it independent from geometry. This view was specially voiced by Peacock[7], who presented a purely symbolical view of Algebra, firstly for students in 1830 in *A Treatise of Algebra*, and then for scientists in 1833, in his « Report on the recent progress and actual state of certain branches of analysis », pronounced at Cambridge for the third meeting of the newly founded *British Association for the Advancement of Science*. With the second generation of this network came on the stage: Augustus de Morgan (1806-1871), Duncan F. Gregory (1813-1844), George Boole (1815-1864), Arthur Cayley (1821-1895) and James S. Sylvester (1814-1897), together with a nebula of less well known mathematicians around them[8]. With these followers, Peacock's symbolical approach was at first expanded as a « calculus of operations », and then diversified in producing new methods and objets, from a calculus on differential operators[9] and logic[10], to matrices and octonions[11]. That production of new objects, beyond quantitative entities, is held as one of the main contributions which impulsed a radical change on the object of Algebra: previously considered as an investigation for a general theory of the resolution of equations, Algebra could then begin to stand as the study of abstract structures[12].

In any case, historians often considered the birth of so modern an approach to algebra in Great-Britain as surprising. At the 18th century and the very beginning of the 19th century, Continental mathematicians were more acknowledged than British ones. And even if the examination for the B. A. degree was concentrated on mathematics in Cambridge, its Geometrical and Newtonian approach seemed to be outrunned, when compared to the algebraical developments of Laplace's *Mécanique Céleste* and *Théorie des Probabilités*. General histories of mathematics regularly referred this faithfulness on Newtonian notation to the quarrel of priority between Newton and Leibniz for the invention of the Calculus. But this reason is too meagre a cause to explain a century of specific development of mathematics in Great-Britain[13]. More precise studies of the Cambridge university context showed that this faithfulness was linked to an attachment to more permanent[14] forms of knowledge. Even if Cambridge educated an elite of gentlemen for the future governing class, it will remain a branch of the Church of England until 1871. In the Anglican universities of Cambridge and Oxford, the obligation of faithful oaths – both in Colleges and University, for undergraduates to obtain degrees as much as for professors to get a chair[15] – structured a traditional conservative way of thinking knowledge as legitimated by cultural values rooted in the past[16].

Of course, facing the Industrial Revolution, what was previously conceived as the educational system of the governing class in order to warrant stability in the whole nation became dangerous manifestations of inertia. Debates in The Edinburgh Review showed how the swift upheavals induced by the industrial world profoundly threatened the ancient equilibrium provided by Anglican universities.

Therefore, the astonishment facing the new symbolical view of algebra sustained first by young Cambridge students stems from a retrospective view of history of mathematics, being only concerned with what announces our present knowledge. As Leo Corry recently urged it, we have to pay attention to the fact that « the image of knowledge »[17] –here that of algebra – was not immediately the one which was developed in the 20th century as investigating abstract structures. The attention on images of knowledge and their different territories – both chronological, geographical, social and conceptual – sheds a new light on some recurring issues on the history of British algebraists, such as: why was there such a long time between the early identification of the properties of a field by De Morgan in 1842, and of a group by Cayley in 1854, and the late development of Abstract Algebra in the 1930s, more than seventy years after? Focusing on the historical background of Peacock's symbolical view of abstraction can help us to answer such questions.

The first part of this paper endeavours to show what was at stake at Cambridge university when this renewed approach to algebra was conceived. Therefore, it will first establish how Peacock's mathematical thought was profoundly involved in his reforming commitment in that institutional and scientific context. Peacock was always close to the Whig policy which echoed utilitarian criticisms on Anglican universities. If he remained a moderate reformer, he nevertheless expressed a constant admiration for the educational institutions born in France with the Political Revolution, and was willing to transform Cambridge from a « seminary of sound learning and religious education » to a « national University ». With such a mind, Peacock was debating on what may be for Cambridge a professional formation in mathematics at a higher level[18]. His whole life was entirely devoted to this reform, until he died as one of the members of the Executive Commission who undertook the first important reformation of Cambridge university in the 1850s[19]. His general purpose was to ground law as objective and rational without religious implications, and so, to express it in such a way that it can support all possible renewals in human practices.

Meanwhile, Peacock also developed an analogous scheme of thought for mathematics. His own philosophy of algebra was sensitive both to inventive mathematical practices and to the deductive form of mathematical reasoning. He tried to establish a mediate path between conflicting trends about mathematics, viewed either as a foundational or as a progressive matter. The « silent perseverance » of his moderate reforming commitment[20] can be perceived nowadays as a « case of creative indecision »[21]. But Peacock remained a Reverend, who took orders in 1817, and became deacon of Ely cathedral in 1839, keeping the Lowndean Professorship of Geometry and Astronomy he obtained only in 1836. And Joan Richards precisely analysed how the religious view of absolute truth was impressed on this network of algebraists[22]. In this paper, we shall focus on the way by which Peacock managed to harmonize this view with his political will to involve new practices – in the university as well as in mathematics. It will be shown that his conceptual view of the genesis of algebra, as built from arithmetical practices, was deeply fostered by Locke's philosophy of language, as still taught at this period in Cambridge university[23]. Locke accepted the major gap between « nature » and the knowledge of nature which can be expressed by the language, and his philosophy afforded an essential role to the symbolical function of representation of language. As we shall see, in the same way as Locke presented the different stages of how the operations of mind worked, Peacock conceived the genesis of operations of algebra, from arithmetical practices to abstract operative laws. In that prospect, Peacock's History of Arithmetic belonged completely to this enterprise, and he will reassert it with the same importance during the 1820s and later, during the 1840s, when A Treatise of Algebra was reedited, as his views had once more to be defended. Examining the language of arithmetic in very numerous countries and periods, Peacock was anxious to value arithmetic amongst mathematicians: he wanted to show the universality of this operative way of thinking, and presented it as the first step of abstraction in the construction of algebra as the « science of general reasoning by symbolical language »[24]. Peacock appeared to his contemporaries as the most philosophical mathematician among all of them[25], and it will be shown how he precisely mobilised all the ressources of rhetorical argumentation to convince his readers of the symbolical function of language as an essential feature in mathematics.

Peacock as a Whig Anglican Algebraist

As Wilkes insisted on[26], the initiatives of this network[27] were not the first attempt to renew Cambridge's curriculum. What was really changing now was a coordinate political determination, as its members attempted to reconcile learned men and practical men, maintaining social cohesion between academics and industrial world, whose science could be the cement. From its beginning, it was essentially organized by a common vision of the necessity of profound unifying reforms, in order to adapt the old institutions to the effects of the Industrial Revolution, and to avoid such a turmoil as the French Revolution. Many of its members could also be found among « the Gentlemen of Science », who managed the British Association for the Advancement of Science during its first twenty years of existence[28]. Its most committed reformers were close to Whigs and Radicals[29]. Peacock and his friends were very much concerned by the gap between the traditional education in the Anglican Universities of Cambridge and Oxford, and the new conceptions of knowledge, fostered by utilitarism and empiricism, firmly praised by the criticisms[30] of the new Whig journal, the Edinburgh Review, born in 1802. They directly worked to understand the epistemological consequences of the Industrial Revolution, and to adapt the old institutions, offering their reflection to the governing class.

1. Peacock and the renewal of scientific institutions

Although Peacock appeared as a moderate man, he constantly manifested a very firm determination in this reforming enterprise. Matriculated as a sizar at Trinity College in 1809, this modest Anglican vicar's son was second wrangler after Herschel for the B. A. degree and second Smith Prize winner in 1813. The letters he wrote to his elder brother working in London Stock Exchange during that whole period soon criticised Radical thinkers such as such as W. Cobbett, J. Horne Tooke, J. Cartwright and Francis Burdett, although he felt very close to them when he entered Trinity[31]. He quickly became more temperate facing also reformers in Cambridge[32]. Nevertheless, Peacock asserted that he will « never cease to exert [himself] to the utmost of the cause of reform »[33]. What he effectively did, both inside and outside Cambridge, maintaining a constant moderate, but straightforward reforming ambition.

Just as he introduced Leibnizian notation in Cambridge examinations, Peacock was directly involved in the creation of the Cambridge Philosophical Society (1819), the Royal Astronomical Society (1820), and later, the British Association for the Advancement of Science (1831), where he often was an officer and referee. These creations were planned to bring a new equilibrium between the learned societies recently founded in the new industrial towns, and the old institutions of knowledge, where severe religious restrictions of admission to the degrees kept out a lot of students. On that ground too, Peacock was constantly extolling a rational faith[34], and worked to loosen the relationship between the religious and the educational roles of Cambridge. Constantly sustaining students against the compulsory attendance to the daily Chapel service[35], he was also one of the four organizers of the public petition submitted to the Parliament in 1834 with sixty-four signatures, asking for the abolition of religious tests in Cambridge examinations[36].

The various Syndicates which were formed in Cambridge in order to develop the study of the different branches of natural philosophy found Peacock among their members. He contributed in that way to the creation of the *Cambridge*

Observatory (1816-1823), the *Pit Press* (1831-1832), the *Fitzwilliam Museum* (1830-1835), and the extension of the Cambridge buildings (1829-1842). Peacock offered the students « the acquisition of accurate knowledge …. not confined to Classical or Mathematics, [but for] other sciences, whether natural, political, or moral »[37]. Arguing for a professional education in Cambridge, Peacock was in a constant opposition with William Whewell (1794-1866), the Master of Trinity from 1841, who conceived Liberal education as general rather than specialized.

With all these commitments, Peacock was in his time a scientist of national importance, who spoke all along his life as an actor of the social transformations to which he was confronted. His letters are those of an advised man, very well informed for instance on economical issues[38]. His correspondence gives evidence of his relationships with the progressive governing class. He was in charge with its sons as a tutor in Trinity College (1814-1836), and so, he was directly in touch with numerous high political or governing personalities during the 1830s, such as Sir G. Grey or M. Spring Rice, later Chancellor of the Exchequer. Lord Melbourne supported him for his nominations both as Lowndean professor in 1836 – against Whewell – and as deacon of Ely in 1839. By carrying on his numerous responsabilites, he served the national institutions or the University[39] rather than the religious Colleges. With his *Observations on the Statutes of the University of Cambridge* (1841), Peacock paved the way for the direct intervention of the Crown in the university affairs, and to its political role in the Cambridge Reform. In this last book, the statutes for permanent functions of the University were carefully distinguished from those for possible local evolutions, and the necessity of their secularisation was insisted on. At the climax of his career, in 1850, he was one of the five members of the Cambridge University Commission chosen by Lord Russell to inquire on the best way to manage the reform, in a way to reinforce the University power on the Colleges. It is worth noting that the Commissioners' Report viewed to involve engineering studies in Cambridge. Despite a strong resistance of the Colleges – Whewell spoke of a violation of the right of property –, the Cambridge University Bill received the Royal agreement in 1855, and an Executive Commission of eight members had to make it effective. Peacock was one of them[40]. But the complete separation between religious and professional education would be imposed only in 1871.

So, this Whig Anglican algebraist partook the general reforming desire for a Liberal education, but if he was much committed to the professionalisation of

University, he was searching the conditions by which the Reform could effectively maintain a common language and permanent law on the local contingent evolutions. Even if the Board of Mathematics, newly founded in 1848, made the Mathematical Tripos a compulsory elementary examination – from which the student could choose different Tripos[41] –, the curriculum and the Senate House Examination for Mathematical Honours were reinforced in mathematics, and the foundational knowledge in Cambridge remained a general one[42].

2. Algebra as a symbolizing process

At the beginning of the 19th century, strong debates took place both in Cambridge, in Oxford, and between the two, in order to determine what would be the permanent foundations of the new inductive sciences, such as physics or political economy. In Oxford, the Noetics raised the same fundamental questions about logic[43] as the Analytics did in Cambridge about mathematics[44]. Strongly impressed by the empiricist criticisms of the Edinburgh Review, the reformers want to grant a place to practices and experiments in the constitution of knowledge. In mathematics, the Analytics and their followers will insist to accept negative and impossible quantities[45], as well as differential operators[46], and to carry out operations on them. Moreover, they were looking for better foundations than just an analogy with operations on arithmetical quantities to legitimate these new computations[47]. The Analytics attempted to make explicit the conditions by which algebra could be expressed really as a science – a science of necessary truths – rather than a judicious notation which simply allowed a mechanical treatment of operations. To secure this view, they strongly asserted the preeminent role of operations, whatever the symbols on which they were applied. So, operations would be defined, no more from their results, but only from their properties as laws of combination.

In that context, Peacock's main goal was to enforce Algebra over Geometry as the fundamental knowledge in the curriculum. For this reason, he first needed to establish that Algebra was not only a tool, coming from the writing of letters in place of numbers with successive extensions of arithmetical practices, but a Science, characterised as such by its deductive rigour and its universality. What he had to make explicit – and even natural – was the logical part of algebra. But what was also essential for this Whig reformer was to preserve the link between algebra and its inventive practices. His project was clearly to banish what appeared as a parrot-fashion transmission of a long established knowledge, which the Senate

House Examination compelled to restitute mechanically for the obtention of the Bachelor of Arts degree, at least for the Wranglers candidates.

The path was difficult to face this twofold requirement, and Peacock elaborated a very specific one. Essentially, he introduced a radical separation between the meaning of the algebraic symbols, and the logic of operations. This radical breaking off constituted the cornerstone of this Symbolical approach[48], for Peacock and his followers. Operations were no more rooted on their possible results, but on their properties. But what was complicated was to preserve too the submission of experience to the theoretical work of mind.

Peacock presented algebra in a constructive epistemological way, as the third step of a historical reconstruction of a genetical process. The first one was Arithmetic, the « science of measure and quantity », a very practical one, but on which he did not insist in this presentation, undoubtedly because of his 1826 paper. The second one was Arithmetical Algebra, where the symbols were « general in their form, but not in their value ». This Arithmetical Algebra was a logical reconstruction[49], where arithmetical quantities such as $(a - b)$ and $\sqrt{(a - b)}$ existed only if $(a \geq b)$. It did not correspond to the « actual state » of Algebra, whose logic often failed because of analogical practices, when the algebraist was confusing the necessary truth – the logic of operations – and the contingent one – the truth of the results[50]. Peacock precisely focused to separate them. He insisted on the computational character of arithmetical operations[51]. The third step was Symbolical Algebra – the sole universal – whose symbols were « general in their form, and in their value ». In Symbolical Algebra, $(a - b)$ and $\sqrt{(a - b)}$ existed symbolically, representing operations without considering any value given to the letters. Peacock exhibited this « language of symbolical reasoning », giving properties by which operations combined the symbols, which Peacock considered as the « laws of combination » on these « arbitrary » symbols[52]. The « business of algebra » was precisely conceived to discover these general forms of algebraical writings.

So, in reading Peacock's work, the relationship between Arithmetical Algebra and Symbolical Algebra was particularly ambiguous, essentially because the first one had a twofold epistemological statute. It worked first as the « science of suggestion » for Symbolical Algebra, in the sense that its results, if they were expressed by general symbols, provided « signs » – in the first meaning of this word – for the mathematician, helping him to guess the general symbolical forms of algebraiucal writings behind them[53]. But it worked too, afterwards, as one of the possible contingent « interpretations » which gave meanings to the symbols of Symbolical Algebra[54]. Consequently, in mathematical reasoning, Arithmetical Algebra was logically subordinate to Symbolical Algebra.

3. A symbolizing process rooted in Locke's philosophy of language

So, this relationship between Arithmetical Algebra and Symbolical Algebra is somewhat troublesome for the modern reader. Peacock pointed out that the laws of combination had to be obtained neither by some extension of those in arithmetic – which induced a change in words meaning – nor by some analogy with them, because analogy was not part of deductive reasoning. He did not prohibit the use of analogy, but he made it the perceptive part of a more essential principle, the famous principle – or law – of permanence of equivalent forms[55]. This principle authorized Peacock not to deduce all the resulting forms in Symbolical Algebra, but only to take them from Arithmetical Algebra, as long as their forms were absolutely general. For instance, a – (b + c) = a – b – c was obtained first in arithmetic, but it does not suppose any numerical values of the symbols; so, it could be considered as a symbolical equality, whose truth is strictly grounded on the laws of combination of symbols. This is the main reason why Peacock did not give an axiomatic deductive presentation of Symbolical Algebra[56]. This double principle asserted:

> *(A): Whatever form is algebraically equivalent to another when expressed in general symbols, must continue to be equivalent, whatever those symbols denote.*
>
> *(B): Converse Proposition: Whatever equivalent form is discoverable in arithmetical algebra considered as the science of suggestion, when the symbols are general in their form, though specific in their value, will continue to be an equivalent form when the symbols are general in their nature as well as their form*[57].

If a retrospective view is not resorted to, this twofold principle cannot be read as any anticipation of a modern inclusion of Arithmetical Algebra in Symbolical Algebra, which a modern reader could be tempted to consider. For a deeper understanding of Peacock's view of Symbolical Algebra, it is essential to underline that he insisted on the two following points: (1) its universality was warranted by operations conceived as laws of combination on arbitrary symbols, so, their results only depended on their properties as laws of combination (2) the meaning of the symbols was a contingent fact: outside their form as a result from operations, it could exist, as for numbers in Arithmetical Algebra, or it could exist not, as for (–1) or $\sqrt{(-1)}$; anyway, it depends on definitions given outside from Symbolical Algebra. The meaning of a symbol considered as too much linked with experience to take part to the universality of algebra. Peacock intended to exhibit algebra as a « purely demonstrative science », which was not at all concerned with the adequacy to the physical world[58].

The arbitrariness of the symbols, the combining character of the operations, and above all, this kind of relationship between demonstration and truth, were also essential features of Locke's *Essay on Human Understanding*. As included in Cambridge curriculum and examinations, Locke's philosophy belonged to the common background of those algebraists in Cambridge. It must be stressed here that Locke already praised algebra, together with moral sciences, for giving the sole universal real truth, because both of them defined freely their words. In consequence, their « real essence » was confounded with their « nominal one »[59]. General ideas and words just concerned the mind, and not at all Nature, or the substance of things, which is unknowable; mathematical propositions are the only ones which could be considered as universal truths, because they were abstract and general. Knowledge is no more than « the perception of the agreement and diagreement between two ideas »[60], and « demonstrative knowledge » was no more than a chain of intermediate ideas which made clear this agreement or disagrement to the mind[61]. So, if the term « operation » is heard with reference to Locke's philosophy, it relates to the way by which the faculties of mind combined ideas and then words[62]. Peacock's vocabulary and methodology are consistent with those of Locke's *Essay.* Already before him, Robert Woodhouse (1773-1827) at the very turn of the century, and Babbage in 1813, by their vocabulary as well as their methodology, resumed Locke's formal conception of demonstration – although none of them, even Peacock, gave his name – as it was in accordance with their search of a theory of invention[63].

So, Peacock is not exactly an anxious or undecided theoretician[64]. He tried to keep together the inventive and the deductive processes of reasoning, but to change their relationship in algebra[65]. Moreover, as a member of the Anglican Church, he intended to secure that operations were not just rules to be applied automatically as a machine would do. He could support them by the operations of mind, and so, he could be sure that algebraical operations still made sense since they were linked to natural faculties of mind, and so, to God's creation. Here stands what allowed Peacock to be satisfied with what is now perceived as a formal view of algebra. The combining processes of the faculties of mind was sufficient for Peacock to endorse both his empiricist faithfulness to arithmetical processes and his theological and teleogical view of truth[66].

PEACOCK'S SYMBOLICAL VIEW OF ARITHMETIC'S HISTORY

Peacock did not need to present the details of the first step of his historical reconstruction of algebra in his 1830s works, since they were previously developed in his extensive paper written for the *Encyclopaedia Metropolitana*. And the analysis of this paper sharpens his philosophical view of algebra as a symbolizing process of operations. Both its writing and its publication took place respectively in 1826 when Peacock developed his specific way of conceiving Algebra, and in 1845 when he had to reinforce it facing the rising influence of Whewell's view of the curriculum in Cambridge.

Peacock's unifying view of mathematics could meet the design of the *Encyclopaedia Metropolitana*. This enterprise was launched by Samuel Coleridge (1772-1834) in 1817. The publication of the 21 volumes and 8 of plates began in the 1820s, but it is difficult to specify the exact date of each paper in it[67]. As a tory propagandist, Coleridge deeply opposed the mechanistic trend of empiricist philosophy, and the danger of plebification and desintegration of knowledge. When the *British Association* was formed in 1831, he advocated the formation of a « 'clerigy', a body of theologians, scholars and men of science, in charge to protect its unity »[68]. Distinguishing between understanding and reason – the last one being the only basis to found wisdom by organizing men's thoughts – Coleridge gave a classification of sciences, built on mathematics as a formal and pure science. With this classification, he preferred a thematic order rather than the alphabetical order of the French *Encyclopédie,* which he condemned for its disorganizing form[69]. Finding here Babbage as one of Coleridge's counsellors, and Peacock as author, enlightened both the scope of this clerigy, and their own commitment in a general investigation for a new unifying view of science.

Peacock's *History of Arithmetic* gathered together his own early researches on languages and arithmetical notations, given at the very first meetings of the *Cambridge Philosophical Society*[70]. Moreover, it bore witness to the continuity of his commitment in supporting a symbolical view of algebra. This impressive paper was not just intended to give full information about arithmetic through ages and human groups. It constituted the first milestone of Peacock's undertaking to conceive Algebra as a pure Symbolical language and to exhibit it as a constructive process based on the natural operations of mind as conceived by Locke. When Peacock gave a genetical presentation of Symbolical Algebra in 1830, he first called up history as supporting his philosophical thesis on the development of mathematics[71]. His main goal was to confer a full acceptance of algebra in the University, and to change its epistemological status, from an art to a science. He wanted algebra to drop the status of a counting tool just for human affairs. So, he investigated what he viewed as the universal aspects of human experience induced by arithmetical practices, in order to establish mathematics as a universal language for other sciences. In this way, Algebra could appear temporarily[72] as the best outcome of the symbolizing process in mathematics. This is why Peacock does not say anything about « theory of numbers »: he is working on the experiential foundations of mathematics, not on theoretical parts of Arithmetic[73].

I would like to show, in this paper, how Peacock used every kind of rhetorical argument in order to convince the reader of the validity of his view. He wrote as an ethnologist, a philologist, an historian and a philosopher of mathematics, giving a reconstructed approach of the development of Algebra. He undertook to persuade his contemporaries that Algebra was the universal language of mathematics, obtained from arithmetical practices and language.

1. Peacock as an ethnologist and a philologist

The paper started with what Peacock called a « metaphysical question ». It « forms a natural introduction to an historical notice of the different methods of numeration, which have been adopted by different nations at different periods of the world » [369 § 2].

The question bore on how a child acquires for instance the idea of the number « four », as distinct from « four horses », or « four cows ». Peacock immediately linked this process to language and to the faculty of abstraction. All along the paper, abstraction is associated with the values of unicity, simplicity, and universality:

> *Abstraction is the creature of language, and without the aid of language, he (the child) will never separate the idea of any number from the qualities of the objects with which it is associated…*

We are thus lead to the distinction of numbers into abstract and concrete, though the abstraction exists merely in the word by which any number is designated, or in the equivalent symbol by which it is represented in different arithmetical systems [369 § 2].

Thanks to numeral words, the child can keep this idea in mind, and he can pronounce it without associating it to a particular thing. So, because words precede signs in the development of arithmetic, Peacock was going to examine, through words, the traces of arithmetical experiences. He was investigating their universal part, hidden by their contingent diversity.

Peacock grounded his remarks on the more recent works about how to understand the origins of language. He covered a much larger scope than British Indologists, adding many recent reviews on foreign numeral languages – such as those in W. von Humboldt, or in diplomats and Jesuit missionaries writings – to Playfair and Colebrooke investigations[74]. *The History of Arithmetic* will be praised by his old friend Herschel as the « most learned history on this subject »[75]. And its structural methodological approach was explicitly fostered by the recent investigations on the origin of languages [371-372 § 8-10]. Following the tracks of comparative grammar[76], he asserted that affinities between different languages must be located by means of grammatical identities rather than from ressemblance between words:

The more philosophical of modern Philologists, indeed, have ceased to regard affinity of the roots as a decisive proof of the affinity of languages; it may arise from the mere mixture of languages, and from the intercourse of the people by whom they are spoken, but it by no means demonstrates them to be of common origin, unless accompanied also by a corresponding affinity of grammatical structure [372 § 10].

Pursuing this idea for numerical languages, Peacock wanted explicitly to prove that, for the arithmetical language, « amongst all nations, practical methods of numeration have preceded the formation of numerical languages » [371 § 8]. Thus, he considered that numeral words depended on operations for counting, which preceeded them. So – but this is a discussed assumption today – he contended that numeral words came directly from these counting methods, which correspond for arithmetic to the grammatical structure just referred to in language.

Arithmetical language was then investigated in order to show that it is firstly established on the perception of numeration methods. Methodologically, Peacock gave a very large series of examples and counter-examples, with a lot of comparison tables, in order to prove his thesis by furnishing a knowledge as probable as possible of the historical and epistemological development of numeral words. His approach was very close to his way of conceiving language in general, and mathematical language in particular: practice came first; people directly played with numeration methods; then they perceived how these methods were organized, and afterwards, expressed them in verbal language, before special symbols were to write them down. So, language was rooted in practices, and the words bore the trace of them.

2. Peacock as a philosopher of mathematics

According to these queries about how to investigate arithmetical words, Peacock chose first to examine languages of « the most primitive and barbarous people », because he considered that they might be less altered than others. Consequently, they could furnish better informations on structural affinities between languages, in spite of some unexplained cases. In fact, Peacock was going to assert that all languages kept the marks of the original decimal scale, and that its universal presence arose from its natural origin: the organization of the human body, with its symmetry and the ten fingers, was considered as the first « natural abacus » [370 § 4]. That first instrument for counting could be praised as universal, because of its « natural » origin, which makes it existing before human thinking itself. Establishing this natural character of the decimal scale was a very strong argument for Peacock in asserting that arithmetical language was universally established:

> *It will be found, upon an examination of the numerical words of different languages, that they have been formed upon regular principles, subordinate to those methods of numeration which have been suggested by nature herself, and which we may suppose to have been more or less practised amongst all primitive people; for in what manner can we account for the very general adoption of the decimal system of notation, and what other origin can we assign to it than the very natural practice of numbering by the fingers on the two hands* [370 § 3].

Referring to Aristotle, and preventing both the mathematician and the philosopher against the « mystical » aspect of very ancient loose analogies – particularly Pythagoreans and Platonists, ones Peacock insisted on the fact that this « natural abacus » constituted the material demonstration of a general law of nature:

> *The universality of the decimal scale proves, according to Aristotle, that its adoption was not accidental, but had its action in some general law of nature…..*[note] *This is a most philosophical principle of reasoning, which leads in the present instance to the correct conclusion, notwithstanding the Pythagorean and Platonic dreams about the perfection and properties of the number* ten, *which are thrown out as conjectures to account otherwise for its general adoption* [383 § 22][77].

The traces of that starting point are first explored through the words which, in different languages, before the ciphers, denominate numbers. Those words indicate « the regular principles by which numeral systems are formed upon ». From § 8 to § 36, Peacock pursued the inquiry in order to prove the preceding assertion through a multitude of peoples: Tibet, China, the Indian Archipelago – essentially Malasian and Javanese –, Celtic languages – including Basque –, and numerous tribes of South and North America, from Polar American to African tribes. He did not forget singular counting specificities, like those expressing 19 or 29 respectively from 20 or 30, which was a rather widespread method in an-hands counting.

Concerning the natural aspect, Peacock showed that if other natural scales of numeration exist, they relate yet to 10: either 5 and 10 are sub-scales of 20, or 5 is a sub-scale of 10. And the scale 20 often comes from people who counted both on their fingers and on their toes. All these scales can be reduced to 10, and have been superseded by the scale 10, either from inside with the own natural progress of knowledge, or « from other nations through commercial intercourse, colonisation, or conquest » [371 § 8]. Here stood precisely for Peacock the distinction between « tribes and nations »: the counting process is natural in the first case, it is more consciously organized in the second one.

In such a way, Peacock considered that scales such as 12 or 2 were established later, as more philosophical ones, issued from a more advanced stage of arithmetical knowledge [371 § 7]: they do not reveal any kind of original practice. More specifically, he devoted several paragraphs to the attention paid by Leibniz to the scale 2, and for the correspondence that he underlined with the hexagrams, which he named « the Cova, or the lineations of Fohi, the founder of the Empire ». But Peacock did not agree with Leibniz's metaphysical interpretation[78] of the 0 and 1, that he called « metaphysical dreaming » [392 § 34], concluding that this scale was already a symbolical arithmetic, but was not suitable for the ordinary wants of every day life. For that reason, it was adopted by a sole genious man, and not by a nation. There, a special attention was immediately given to Chinese numeral words, because in that case, as Peacock wrote: « Chinese expressions for numerals are in all cases symbolical ». They are simply specific keys of the ideographic language [376 §13].

However, Peacock emphasized early the « two great difficulties attending the invention of our system of decimal notation »: what he named « the local value » (nowadays « positional value ») and the invention of zero [374 §11]. Then he considered that the numerical language of Thibet already overcomed the first one, and so, probably, « we are indebted to this country for our system of arithmetical notation ». And so, Peacock considered the Indian numeral system as the first complete invention of decimal system. Simultaneously, he followed the philologists in ascribing to the Sanskrit language the origin of the classical language of Europe[79]:

> The intimate analogy in the grammatical structure, and in many of the roots of the classical language of Europe with the Sanskrit, combined with the evidence furnished by historical and other monuments, point out the East as the origin of those tribes, whose progress to the West was attended by civilisation and empire, and amongst whom the powers of the human mind have received their highest degree of development [372 § 12].

Finally, Peacock concluded on § 33 the demonstration he began on § 8: that the natural scales are founded on the decimal one, and that: « The natural scales of numeration alone have ever met with general adoption » [371 § 8, emphasis mine]. It is noteworthy that Peacock did not distinguish clearly between numerals and their representation all along this overview, probably because he considered that both of them expressed this same natural and universal decimal character.

3. Peacock as a Whig actor in society

At this point of the demonstration, the arguments on the unification of scales of numeration met a political argument that Peacock already employed for the unification of languages[80], insisting on the political role of the nations to sustain both of them, claiming that : « The natural scales only are national » [391 § 33, emphasis mine].

Already when speaking about Chinese and Indian numeral systems, Peacock underlined how « nations » played an essential part in their development.

He used the existence of an organized society to sustain an argument on utility, which was a very widespread argument for the utilitarian trend to which he was linked. For him, the existence of a useful, powerful system of numeration was closely linked to that of civilization and of a strong state, for example about the developing needs they sustained for writing large numbers, even if the corresponding nouns did not always exist [377 § 16]. About the Aztecks, Mexicans, Muyscas and Peruvians for instance, Peacock clearly linked the perfection of their numeral systems and the existence of organized governements, even if they expressed numbers with the vicenary scale – the base 20 – which, he assessed, was derived from the base 10:

> *The Mexicans, Muyscas and Peruvians constitute the only three nations of Ancient America, who possessed government regularly organized, and who had made considerable progress in many of the arts of civilized life, in architecture, sculpture, and painting. They were the only peoples, in short, in that vast continent, who could be considered as possessing literary or historical monuments. On this account alone their numeral systems would merit very particular attention; but still more so from their perfect development. The first presents the most complete example that we possess of the vicenary scale, with the quinary and denary subordinate to it. The second, of the same scale, with the denary alone subordinate to it; whilst the third, or Peruvian, is strictly denary, and is equally remarkable for its great extent and regularity of construction* [389 § 29].

Here can also be stressed the special care with which Peacock spoke of the Peruvian Quipus, which constitute for him a very perfect and material representation of numbers, in a decimal scale, « incomparably superior to those of any other American nation » [390 § 290]. Quipus form a system of knots on different coloured strings. Indeed, they allow the recording of numbers and also – Peacock wrongly thought – the practice of arithmetical operations with a particular rapidity and accuracy. Anyway, they constitute an excellent administrative tool reserved to their guardians[81]. Here was recognisable the insistance of the reforming network, particularly sensible in Babbage, Peacock and De Morgan positions – as members of the Decimal Commission, but also in their public undertakings – on the necessary commitment of national organizations in the progress of knowledge[82].

After his investigation of numeral words, Peacock turned to the invention of numeral symbols, which he named « symbolical arithmetic ». That one is no more linked with the most primitive people, but with historical periods, and organized societies. Once more, his presentation was directly linked with that of operations, not only the four elementary operations of arithmetic, but also the extraction of square and other roots of numbers. He presented all of them with numerous examples.

Peacock began this presentation with a very long paragraph on the arithmetical notation of the Greeks, because « they cultivated the sciences for the greatest success » [394 § 38]. He noted Delambre's disappointment not to see them developing the decimal notation: announcing his own way of thinking operations, he underlined the too strong attachment of the Greeks to their alphabetical notation [405 § 41]. Nevertheless, in order to stand out from Delambre, and to show that Greek inventivity was not absent, Peacock appealed to Archimedes' *Arenary*, which gave the means to overtake the limitations of the initial system. On this point, he considered Stifel and Stevin contributions in the 16th century as an extension of this work of Archimedes, and insisted on the fact that progress is not there if the utility of notation is not socially perceived:

> *There are many of the artifices of notation employed in this work, which if pursued and properly generalized, would have given increased symmetry as well as extent to their symbolical Arithmetic..… The only reason which can easily be assigned why this extension of their notation had not been generally adopted for all the symbols, when once applied to those of the nine digits, appears to have been, that as they merely proposed by it, in the first instance, to make their notation coextensive with the terms of their numeral language, they paused when that objet was effected; and, however simple its extension to all the other symbols may have been, it was not likely to be adopted when the utility of it was not felt; the advantage indeed of a simple and expressive notation addressed to the eye, as distinct from language, were in no respect understood by the ancient geometers; and it is only in modern times that the power of symbolical language have been completely appreciated* [397 § 38]

On the same way, among the Greeks, Peacock payed a special attention to Ptolemy, because of his preference for the sexagesimal notation. According to Peacock, Ptolemy's choice was clearly linked with astronomy, because it allowed the divisions of the circle to be « nearly equal to the days in the year » [401 § 39]. Peacock explained how it was essentially used to avoid fractions, because of the numerous factors of 60. Therefore, this system too was considered as a refined state of development of the numeral systems, which was largely used before the introduction of the « Hindu notation »[83]. Contrary to Theon of

Alexandria's assertion, Peacock dared to consider that the invention of sexagesimal arithmetics preceded that of Ptolemy, concluding:

> *Whoever, however, was the author, it must be considered as the greatest improvement in the science of calculation which preceded the introduction of the Hindoo notation; it enables astronomers at once to get rid of fractions, the treatment of which in their ordinary arithmetic, was so extremely embarrassing: and enables them to extend their approximations, particularly in the construction of tables, to any required degree of accuracy* [401 § 39].

Following then a chronological order, and watchful to the role of nations, Peacock attributed the importance acquired by Roman ciphers to the domination of Romans. Still underlining the above parallel between the low state of development of different nations and the lack of notation, Peacock gave a large place to what he named « Palpable Arithmetic », which practice authorized operations when fitted notations were missing. He presented there all kinds of abacus: Roman abacus, Chinese Swan Pan, *Logistica Tabula* (tablet strewed by sand), and the Greek Abacus [407-410 § 51-57]. He insisted on the general use of such counters in Europe until the end of the 15th century in Italy, and until the 16th and 17th century in France and England, mentioning notably Saunderson's calculating board for the blinds, or Napier's multiplicative rods. Peacock alluded here to Leibniz's arithmetical machine and to Babbage's Difference Engine as extensions of this trend of counting:

> *The existence of systems of symbolical Arithmetic implies some considerable progress in the arts of life; and we, consequently, cannot expect that such systems should be numerous, particularly when we consider how few are the nations with whom civilization had been of native growth* [407 § 51]

Following his view on utility, Peacock pointed out treatises such as *Arithmetic, or the Ground of Arts* (1540) of Robert Recorde, and *Arithmetica Practica* (1662) of Gaspari Schotti, because they indicated the operating rules of this « Calcular Arithmetic », which was so much in use among the merchants that it was named *Arithmetica Mercatoria.* Merchants were specially praised there for their organizing action. Peacock tackled the argument of utility not only with an historical point of view, but with a technical one. It can be seen there that he was not really interested by enquiring on the origins on Arithmetic, but only on how history testifies to the natural and universal character of Arithmetic, in order to legitimate the importance of Arithmetic as a natural foundation of Algebra, a language with special notation. According to Peacock, the importance paid to the « Hindu » decimal notation came essentially from the fact that it had superseded the preceding ones, and that it authorized plainly the transition from palpable arithmetic to written arithmetic.

As Dhruv Raina will analyze it in his following paper, what is essential to Peacock is, firstly, to assert that the « Hindu » Arithmetic is at least as ancient as Diophantus' one[84] [413 § 66], and secondly, to show that its notation was adopted by the Arabs from « Mohammed ben Musa, the Khuwarezmite », his almost contemporary al Kindi, and the astronomer Ibn Younis [413 § 67], and transmitted by them, as early as the 10th century, in all the countries where the Arabic language was known, specially in Spain, and to Europe. And he devoted special historical attention to the conditions of that transmission.

4. Peacock as a historian

From there, Peacock worked essentially as a genuine historian of mathematics[85], worrying to write history with other glasses than those of his own time, providing even appropriate criteria to do so. His methodology bears witness to the birth of the history of mathematics as a new discipline, independent of traditional reviews of diplomats and missionaries, whose investigations he analysed yet[86].

Firstly, Peacock fostered his presentation by a large range of examples, very carefully choosen in order to precise the whole scale of numeration and operation methods. In that way, he opened the way for the reader to get his own view, giving a complete access to his own sources, discussing manuscripts[87] as well as original books. For instance, when he tried to determine as exactly as possible the dates when the Arabic ciphers replaced the Roman ones, Peacock refused to observe just the datations, and preferred to investigate the calendar computations through *Computatio Ecclesiastico*, which was a printed book. Studying John Wallis (1616-1703), who is still well known for his interest in ancient and Arabic texts, Peacock put him on his guard against the possible confusion between the moment when symbols are written and the moment when they are really used, because of the delay between the real practice and the work of the copist, which occurred before the development of printing. Peacock discussed manuscript dates, places where the manuscripts were found, and did not hesitate to dispute other scholars' views, such as Delambre's about the Greeks. For instance, on the first role of Gerbert d'Aurillac in the introduction of the Arabic ciphers from Spain to France, Peacock was not so convinced as Colebrooke by Wallis' arguments. He discussed also the precise period when Leonardo Pisano (*Liber Abaci* in 1220) was living and working between Egypt; « Barbary », Syria, Greece and Sicily. Moreover, as a Whig reformer, Peacock underlined very carefully the inertia of traditions in this process of adopting the decimals, in educated colleges as well as on the market place. The essential point was to conclude that the « Indian mode of computation » was introduced in Spain[88] earlier than in Italy, even if its diffusion was not general, and depended on the places where it was used.

What was more important for Peacock was the extension of arithmetical practices to various types of trades. He studied it in *La Disme* (1590) from Simon Stevin, which marked the introduction of the decimal system in merchant practices [440 § 136]. His insistance on Stevin's work reveals once more Peacock's perspective, and it is very remarkable[89], whereas this work is alas not so much praised generally in classical histories of mathematics nowadays, regarding its role for the extension the idea of number. Peacock considered *La Disme* introduced an important progress in universality and abstraction, in a time when arithmetical practices were often linked with concrete numbers, and where the subdivisions of systems of measures were established relatively to the human body, and therefore to hand-made labour. On the contrary, Peacock asserted that if Spain was important for the adoption of this decimal system, it was confined to the translation of the astronomical texts from Arabic in Latin, before that the « contests which distracted this country » concluded with the « final expulsion of the Moors » [415 § 70]. Spain was then relayed by Italy, which played a more important role in realising the advices of Stevin.

Here, Peacock insisted on the role in return of these new practices on the development of arithmetic, and more precisely on the role of commercial and banking practices in improving arithmetical operations, explaining them with great details. Bills of exchange and book-keeping received a special attention. While their practice tended to make their users considered as usurers, Peacock worked to establish the contrary and to restore their social dignity. The reforming commitment of Peacock appeared clearly here. As a Whig whose eldest brother worked in Stock Exchange[90], Peacock was particularly concerned with the new ways by which mathematics can govern economy, in Quattrocento Italy as well as in Great-Britain at his time:

> *The Tuscans generally, and the Florentines in particular, whose city was the cradle of the literature and arts of the XIIIth and XIVth centuries, were celebrated for their knowledge of Arithmetic: the method of book-keeping, which is called especially Italian, was invented by them; and the operations of Arithmetic, which were so necessary to the proper conduct of their extensive commerce, appear to have been cultivated and improved by them with particular care; to them we are indebted for our present processes for the multiplication and division of whole numbers, and also for the formal introduction into books of Arithmetic, under distinct heads, of questions in the single and double rule of three, loss and gain, fellowship, exchange, simple interest, discount, compound interest, and so on; in short, we find in those books, every evidence of the early maturity of this science, and of its diligent cultivation; and all these considerations combine to show that the Italians were in familiar possession of Algorithm long before the other nations of Europe* [414-415 § 70]

The word « algorithm » was here specially analysed by Peacock, who observed the transformation of its meaning, once more by way of mathematical practice. He even laughed at Stifel who ignored the Arabic origin of this word, when he insisted on the fact that it was born in the same time and at the same place as the word « Algebra »:

> *The term algorithm, which originally meant the notation by nine figures and zero, subsequently received a much more extensive signification, and was applied to denote any species of notation, whatever for the purpose of expressing the assigned relations of numbers or quantities to each other* [438 § 132].

And Peacock devoted a long place to the improvement of the approximation processes that the decimal system brought, thanks to the decimal fractions, and to the dot notation. Peacock's conclusion on this point was really essential to his design about notation:

> *In general, however, it may be remarked, that the invention of a distinct, expressive, and comprehensive notation, is the last step which is taken in the improvement of analytical and other sciences; and it is only when the complexity of the relations which are sought to be expressed in a problem is so great as to surpass the powers of language, that we find such expedients of notation resorted to, or their importance properly estimated* [438 § 130].

My conclusive point would bear on the long detailed review that Peacock presented about the new French measures system, in other words the adoption of the metrical system of weights and measures during the French Revolution, from « purely philosophical principles », which were in accordance with his Whig's position on the progressive unifying power of nations[91]. It reflected Peacock's profound admiration of the new French institutions born with the Revolution:

> *If ever an opportunity presented itself for the establishment of a system of weights and measures upon perfectly philosophical principles, it undoubtedly occurred in the early part of the French revolution, when the entire subversion of all the old establishments, and the hatred of all associations connected with them, had created a passion for universal change* [446 § 171].

This review began with the measure of the pendulum vibrating seconds by Richer in 1671, the discussions of Cassini, and De la Condamine during the 18th century, and finally, the decisions of the revolutionary commission in 1794 and 1798. If Peacock did not hide the difficulties of this adoption, and the disadvantages that it supports for the centesimal division of the quadrant, he insisted on the universal aspect of the new unity, quoting in French this part of the report of the 10 Prairial 1798 to the two councils of the legislative body:

> *Cette unité, tirée du plus grand et du plus invariable des corps que l'homme puisse mesurer, a l'avantage de ne pas différer considérablement de la demi-toise et des plusieurs autres mesures usitées dans les différens pays: elle ne choque point l'opinion commune. Elle offre un aspect qui n'est pas sans intérêt. Il y a quelque plaisir pour un père de famille à pouvoir se dire: "Le champ qui fait subsister mes enfans est une telle portion du globe. Je suis dans cette proportion conpropriétaire du monde"* [448 § 174]

Moreover, this adoption enabled the union of operations on concrete numbers and operations on abstract numbers, contributing to the unification of the numerical realm. Once more, and as Babbage will later do for his engines, Peacock praised the political intervention to help the unification of knowledge.

Conclusion

This History of Arithmetic can really stand as a very well documented reference about what was known and discussed about the historical construction of arithmetic as a human undertaking. Nevertheless, in spite of his ethnological, philological, philosophical and historical enlarged positions, it is clear that his own starting conceptions about the construction of knowledge through language lead him to some limitations. For example, Peacock did not try to recompose the own processes of Indian and Arabic algorithms, notably for the rule of approximation of surd numbers, or for the rule of alligation and of position. In several places, he substitutes for them their algebraical outcome [436 § 125 & 463 § 238], what appears nowadays as a crucial aspect of a recurring position. His representation of algebra helps us to understand his comparative and philological approach to different numeral languages, and the respective places afforded particularly to Arabic and Indian arithmetic in the presentation of contemporaneous knowledge on numbers. As an Anglican Whig mathematician Peacock conceived these processes both as natural, and supported by the constitution of nations, writing a really « social study » of arithmetic, impressed with utilitarianist values on the organzing role of State, even on structuring knowledge. Consequently, if Peacock's view maintained some recurring interpretation of arithmetical processes, it introduced a really constructivist one, notwithstanding the faculties of mind.

Peacock's main goal in this paper was to bring dignity to the arithmetical practices – which was truly new in academic circles – so as to connect together the scholarly knowledge and the tradesman's knowledge, by showing their universal sources, grounding them on the common experience of all people facing the world everywhere. Peacock's representation of mathematics committed an empiricist conception of knowledge, even a mathematical one: the development of algebra is traced from arithmetical practices founded on the decimal system of numeration, and this one is conceived as a natural, and therefore as an universal one. Such a knowledge was established from experience and organized by successive steps of symbolization, until Symbolical Algebra, and that view was deeply rooted in Locke's philosophy. But Locke's empiricism was still a very moderate empiricism, where the operative faculties of mind still stand as innate. In such a way, this History of Arithmetic stands as an integral part of Peacock's whole prospect of a very particular view of Algebra, which attempted to hold together contingent mathematical practices and universal necessary truth of deductive mathematical reasoning. Peacock can be praised for this powerful attempt: with his Symbolical Algebra, he tried to solve philosophically the difficult issue of the nature of algebra, with the mathematical means of the first half of the 19th century.

Acknowledgments

We would like to thank Karine Chemla and REHSEIS for their constant financial and moral support all along the preparation of our papers. Josipa Petrunic and Agathe Keller worked a lot on the English way to write correctly this paper.

Notes & References

Special notice: *references from Peacock's* **«** *History of Arithmetic* **»**, *given as [(x) § (y)] means* [Peacock, 1845, p. (x) § (y)].

A complete bibliography of Peacock's works and of this Peacock's paper is given at the end of the paper.

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3. W. F. K. Cannon, 1964, « Scientists and Broadchurchmen: An Early Intellectual Network », *Journal of British Studies*, IV, no. 1, 65-88.

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5. In 1816, Babbage, Herschel and Peacock translated Lacroix's *Elementary Treatise of Integral and Differential Calculus*, with abundant notes by Peacock, and each of them prepared a special *Collection of Examples* in 1820. This need for new textbooks was abundantly referred to by these three students in their correspondence. Cf. Royal Society Library. *Herschel Manuscripts*, Hs.2.69, Herschel to Babbage: 24.12.1816; Hs.13.246, Peacock to Herschel: 13.11.1816; Hs.13.249, Peacock to Herschel: 17.03.1817; Hs.13.250, Peacock to Herschel: 30.05.1817.

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15. These oaths induced such an interiorisation of this traditional conservative way of thinking that any attempt to reform the system could be accused to be a perjury.

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24. Peacock, 1830, *op. cit.* note 7, 1.

25. Herschel, 1858, *op. cit.* note 19.

26. M. V. Wilkes, « Herschel, Peacock, Babbage and the Development of the Cambridge Curriculum », *Notes and Records of the Royal Society of London*, 44 (1990) 205-19.

27. As Ivor Grattan-Guinness indicated to me in 1998 in Wuhan (China), « network » (Cannon's denomination) is a better term than « school » (Novy's denomination) to designate this group of algebraists. « Network » does not mean a unique Master and his followers, but rather several authors working on algebra as marked by the symbolical function of language, but with different directions of research inside this movement.

28. J. Morrell and A. Thackray, *Gentlemen of Science, Early Years of the British Association for the Advancement of Science*, Oxford, 1981.

29. C. Smith & M. N. Wise, 1989, *Energy and Empire: A Biographical Study of Lord Kelvin*, Cambridge, Cambridge University Press, ch. 6.

30. J. Playfair, « Review of Laplace's *Traité de Mécanique Céleste* », *Ed. Review*, 11 (1808) 249-84; (Anonymous), Review: « A Reply to the Calumnies of the Edinburgh Review against Oxford; containing an Account of Studies pursued in that University. Oxford », *Edinburgh Review*, 16 (1810) 158-87.

31. Trinity College Library, *Peacock Manuscripts*, Peacock to his brother William, 21.08.1810, P 3[7]; 30.09.1810, P 3[8].

32. Trinity College Library, *Peacock Manuscripts,* Peacock to his brother William, 02.03.1811, P 3[10].

33. Royal Society Library, *Herschel Correspondence*, Hs.13.249, Peacock to Herschel: 17.03.1817.

34. For instance, travelling in Italy, he considered Catholic manifestations of faith, such as processions or adoration of Turin's shroud, as fanatic attitudes. Trinity College Library, *Peacock Manuscripts,* Peacock to his father, 27.07.1816, P 3[43]; Peacock to his sister, 28.07.1839, P 3[89].

35. Anonymous, 1858, « The late Dean of Ely », *Cambridge Independant Press,* from *The Times,* 13.11.1858, p. 6.

36. This action failed in House of Commons. Anonymous, 1845, *Cambridge Chronicle,* 28.03.1834. Anonymous, 1859, *Cambridge Independant Press,* 20.11.1859, p. 7. M.-J. Durand(-Richard), 1985, « George Peacock (1791-1858): La Synthèse Algébrique comme loi symbolique dans l'Angleterre des Réformes », *Thèse pour le doctorat de l'E.H.E.S.S.*, (1830) pp. 236-239.

37. G. Peacock, *Observations upon the Report made by a Syndicate appointed to confer with the Architects who were desired to furnish the University with designs for a new Library,* Cambridge, p. 13.

38. In Paris during the political Revolution of 1830, he appreciated the new Monarchy, as he wrote: *I yesterday saw the Chamber of Deputies proceed to the Palais Royal to offer the crown to the Duc d'Orléans: .... it is however quite clear that he unites the votes of all the well informed classes of men in France and that his elevation will meet with universal acquiescence at least if not with universal approbation... The French justly dread a civil war and it is surprizing to observe the readings with which they are ... to sacrifice their personal opinions and wishes to the cause of public union and peace*. Meanwhile, Peacock informed his brother of the transactions in Paris Stock Exchange. Trinity College Library, *Peacock Manuscripts,* Peacock to his brother William, 01.08.1830, P 3[70]; 05.08.1830, P 3[71]; s.d. received on the 10.08.1830, P 3[72]. In 1846, Peacock also published *Upon the Probable Influence of the Corn Laws upon the Trade of Corn,* and, travelling in Madeira for his health in 1850, « A Review on the State of Agriculture » in that island. As deacon of Ely, Peacock also exerted his administration according to three essential axes of the Whig program: health, justice and education.

39. Peacock was secretary of Cambridge University Chancellor, the duke of Gloucester (1831), and then, secretary (1837-39) and president (1844) of the British Association.

40. Durand(-Richard), *op. cit.* note 36, (1985) pp. 192-276, pp. 315-344. A similar strand of reforms took place in Oxford at the same period.

41. After the elementary Mathematical Tripos, these different Tripos were: the Classical Tripos, the Moral Tripos, The Natural Sciences Tripos, and the Mathematical Honours.

42. M.-J. Durand-Richard, « L'Ecole Algébrique Anglaise: les conditions conceptuelles et institutionnelles d'un calcul symbolique comme fondement de la connaissance », in (eds) Goldstein, C., Gray, J., Ritter, J., *L'Europe Mathematique - Mythes, histoires, identité. Mathematical Europe - Myth, History, Identity,* Paris, Eds M.S. (1996) 445-77. A. Warwick, 2003, *Masters of Theory, Cambridge and the Rise of Mathematical Physics*, Chicago and London, The University of Chicago Press.

43. P. Corsi, 1988, *The heritage of Dugald Stewart: Oxford Philosophy and the method of political economy,* Firenze, 1988.

44. Durand-Richard, 1996, *op. cit.* note 41.

45. H. Pycior, 1997, *Symbols, Impossible Numbers, and Geometric Entanglements*, Cambridge.

46. M. Panteki, 2000, « The Mathematical Background of George Boole's *Mathematical Analysis of Logic* », in *Anthology on Boole* (ed.) James Gasser, Waterloo (1847) 167-212.

47. Analogy was currently used since the early developments of algebra, to legitimize operations of new objects. John Playfair (1748-1819) for instance recently used geometrical analogies. J. Playfair, « On the Arithmetic of Impossible Quantities », P*hilosophical Transactions of the Royal Society*, 68 (1778) 318-343.

48. Peacock already referred explicitly to syntax of language at the end of his « History of Arithmetic » [481 § 280].

49. This logical reconstruction of Arithmetical Algebra stood as an attention paid to the early criticism of algebra by the Radical William Frend (1757-1841) – the future father-in-law of De Morgan –, who asked to exclude negative and impossible quantites from algebra, because they did not correspond to any experience, and introduced logical contradictions since the meaning of operations was therefore changing. W. Frend, *The Principles of Algebra,* Cambridge, 1796.

50. For Peacock, most of the paradoxes that existed in algebra, such as those about the logarithms of negative quantities, came from the confusion between contingent and necessary truths, or between the arithmetical value and the symbolical representation of the letters.

51. For instance, in his new edition of the *Treatise* in 1842 and 1845, where he separated in two volumes Arithmetical and Symbolical algebras, Peacock insisted particularly on « interminable quotients », when division does not stop, and quotients became decimal numbers. Essentially, from his symbolical view on operations, this case did not make difference with a division where the exact quoitient is a integer. Clearly, at this time, Peacock needed to reinforce his position in front of Whewell's one in his *History of Inductive Sciences* (1837) and *Philosophy of Inductive Sciences* (1840). G. Peacock, 1842-45, *Treatise of Algebra*, 2nd ed. Cambridge, II, 26.

52. Amongst the « assumptions which determine the symbolical character and relation » of addition and substraction, Peacock wrote: « They are the inverse of each other. … When different operations are performed or indicated, it is indifferent in what order they succeed each other ». Peacock, *op. cit.* note 7 (1833) 196-197.

53. F. Duchesneau, *L'empirisme de Locke,* La Haye, (1973) 200-202.

54. These meanings or interpretations of symbols could be numbers, or forces, or anything else. But they were not at all necessary to the symbolical existence of algebraical forms.

55. Peacock, *op. cit.* note 7 (1830) 108.

56. Durand (-Richard), 1985, *op. cit.* note 36. Fisch, *op. cit.* note 21 (1994) 264.

57. Peacock, *op. cit.* note 7 (1833) 194.

58. Peacock, *op. cit.* note 7 (1833) 187.

59. Durand(-Richard), 1991, *op. cit.* note 23. J. Locke, 1694, *Essay on Human Understanding*, 2nd ed., London, 1694, IV.5.2.

60. Locke, 1694, *op. cit.* note 59, IV.1.2.

61. Locke, 1694, *op. cit.* note 59, IV.1.9.

62. *The other Fountain* [the first one being sensations conveyed into the mind from the perceptions of external objects*], from which experience furnisheth the Understanding with ideas is the Perception of the Operations of our own Minds within us, as it is employed about the Ideas it has got; which Operations, when the Soul comes to reflect on, and consider, do furnish the Understanding with another set of Ideas, which could not be had from things without.* Locke, 1694, *op. cit*. note 59, II.1.4.

63. M.-J. Durand-Richard, « Révolution industrielle : logique et signification de l'opératoire », *Mélanges en l'honneur d'Ernest Coumet*, Paris, n° spécial de la *Revue de Synthèse*, « Histoire des jeux, jeux de l'histoire », T. 122, 4e S. n° 2-3-4, avril-décembre, Centre International de Synthèse, Albin Michel, (2001) 319-346.

64. Fisch, 1999, *op. cit.,* note 21.

65. Peacock intended to keep in mind that algebraical practices could produce new objects such as "(–c), but, whatsoever be the way by which it was first introduced orinterpreted – geometrically for instance –, it would be legitimated in symbolical algebra by a deductive process such as:

$$\sqrt{a-(b+c)} = \sqrt{a-b-c} \quad \text{when } a = b.$$

66. Richards, 1980, *op. cit.* note 22.

67. I. Grattan-Guinness, « Mathematics and mathematical physics from Cambridge, 1815-40: a survey of the achievements and of the French influences », in (ed.) P. M. Harman, *Wranglers and Physicists: Studies on Cambridge physics in the nineteenth century,* Manchester, Manchester University Press (1985) 84-111.

68. R. Yeo, 1993, *Defining science, William Whewell. Natural knowledge and public debate in early Victorian Britain*, Cambridge, Cambridge University Press, 44.

69. S. Coleridge, « General Introduction, or Preliminary Treatise on Method », in *Encyclopædia Metropolitana*. (ed. Smedley and Rose) London, 1 (1845) 1-43.

70. When Peacock speaks of « the present year 1826 » in his *History of Arithmetic*, he corroborates this point [note p. 413 § 66], as well as in the Appendix to his paper. Yet, some bibliographical references indicate that Peacock completed it after 1826. The *Minutes of the Cambridge Philosophical Society* for 1826 and 1827 gives the titles – but alas only the titles – of Peacock's unpublished communications: « on Greek arithmetical notation » (27.01.1826, 13.03.1821), « On the Origin of Arabic Numerals , and the date of their Introduction in Europe » (10.04.1826, 24.04.1826), « On the numerals of the South American Languages » (11.12.1826), « On the Discoveries recently made on the subject of the Hieroglyphics » (12.03.1827,21.05.1827), « Account of the Representations occurring in Egyptian Monuments, of the Duties of that Country, and of the funeral Rituals » (07.02.1828). There is no trace of them in Peacock's papers at Cambridge Trinity College Library. Durand(-Richard), *op. cit.* note 36 (1985) 244-245.

71. The inequality in the three parts of the paper pointed out to the foundational turn of the project: 114 pages beared on the historical notice of the different methods of numeration, 22 pages on the operations on abstract numbers, and 19 pages on the operations on concrete numbers.

72. Some years later, in 1836, De Morgan would present his « Calculus of Functions » in the same *Encyclopædia Metropolitana*, as the next step in this symbolisation process of mind operations. A. De Morgan, 1845, « Calculus of functions », *Encyclopaedia Metropolitana*, London, [1836] II, 303-92.

73. There was a specific paper on « theory of numbers » in the *Encyclopædia Metropolitana*, written by Peter Barlow (1776-1862). In his History of arithmetic, Peacock precised that he did not develop operating practices when they are founded on specific algebraic knowledge, and moreover, he attributes the confusion of some methods in algebra to the ignorance of their underlying knowledge [437 § 128-129].

74. See Dhruv Raina's paper, and Peacock's bibliography of his « History of Arithmetic », in the same issue of this Journal.

75. Herschel, *op. cit.* note 19.

76. Comparative grammar was then one of the favourite subjects of *The Apostles* in Cambridge, and Peacock was close to them. He also exchanged with W.D. Conybeare on that subject. *Trinity College Library*, Cambridge, correspondence of Peacock, P.1[88].

77. Peacock firstly gave this quotation by Thomas Herbert in Greek.

78. *As unity was considered the symbol of the Deity, this formation of all numbers from zero and unity was considered in that age of metaphysical dreaming, as an apt image of the creation of the world by God from chaos* [392 § 34]

79. But Peacock did not try to solve the question of the unique or multiple origin of all languages. What is more interesting for him are the conditions of development of arithmetical language. Only in that case, he can decide of the unique origin, because of the « natural abacus ».

80. This argument also met Peacock's political position in Cambridge, where he urged the unifying role of the Crown at Cambridge, in supporting University against the Colleges.

81. What Peacock forgot to indicate, is that these Quipus are much more remarkable because they were found in societies without scripture. Investigations on Quipus still question researchers today.

82. Babbage's *Reflections on the Decline of Science in England, and some of its Causes* (1830) and Peacock's *Observations on the statutes of the University of Cambridge* (1841) can be specially quoted on that topic.

83. « Hindu » is the modern writing of the term « Hindoo » used by Peacock. It is written here between quotation marks, because nowadays, it could have a religious connotation, and seems to exclude non-Hindu Indians. Besides, using the term « Indian » could seem to exclude geographical places as Pakistan, Bengladesh or Sri Lanka.

84. Perhaps even much earlier, but in any case, « long before the Persians, Arabs, or any western countries » [413 § 6§].

85. Even if this historical analysis was intended to serve a specific Whig view of history.

86. Peacok will be also more generally concerned by history. When he becomes deacon of Ely, from 1839 to his death, he managed the restoration of the cathedral, and studied documents from 1374 on its history, newly found during the works. *Cambridge University Library*. Peacock's correspondence with R. Willis. Add. Ms. 5026 ff 24-30.

87. Peacock referred to manuscripts of the British Library as well as those of the Bodleian Library in Oxford.

88. If Peacock doubted of Gerbert's role, he asserted that the « Indian mode of computation » was present in Spain in 1136, in a translation of Ptolemy, but that it was not in use on the calendars before the 16th century [413 § 68].

89. J.T. Merz praised Peacock's review of Stevinius as the best one at the end on the XIXth century. J. T. Merz, *An History of Scientific European Thought in the nineteenth century*, 4 vols, Gloucester (1898) 1976, **I**, 645.

90. Peacock's correspondence with his brother William, between1807 to 1838. Trinity College Library, Cambridge, Peacock's papers, P 3^{x}. Also in Durand(-Richard), *op. cit.* note 36 (1985) 543-555.

91. From § 150 to 170, Peacock shew at first the enormous diversity of ancient systems of weights and measures