SEVENTH FRENCH PHILOSOPHY OF MATHEMATICS WORKSHOP

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Reference and Analysis in Analytic Number Theory

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Analytic number theory uses real and complex analysis to study problems about the prime numbers. In the 18^{th} century Euler uses analysis to re-prove the infinity of the prime numbers, and his approach then inspires Dirichlet and Riemann in the 19^{th} century. This path of research leads to important results about the distribution of primes in the work of Hadamard and de la Vallé-Poussin around 1900, because the reduction of questions about integers to questions about the divergence and convergence of certain series, offers much more powerful and flexible techniques than algebra in many cases. Conversely, once this habit of transposing problems upstairs to real and complex analysis is established, problems that arise originally in the infinitesimal calculus turn out to have important consequences for the study of the integers: the study of elliptic functions begins at the end of the seventeenth century in connection with the mathematical modeling of the pendulum, which entails finding a way to determine the arc length of an ellipse. The eighteenth century tendency to study problems of number theory analytically, embedding the study of the integers in the study of real-valued functions, and the nineteenth century tendency to embed real analysis in complex analysis, provides an important background for understanding the reduction of Fermat's Last Theorem to the Taniyama-Shimura Conjecture. Important problem-reductions combine, juxtapose and even superpose discourses that are more concerned with analysis, and discourses that are more concerned with reference. Wiles' proof is not only about the integers and rational numbers; it is at the same time concerned with much more 'abstract' and indeed somewhat ambiguous and polyvalent objects, elliptic curves and modular forms. So for example at the culmination of Wiles' proof, where analysis has invoked cohomology theory, L-theory, representation theory, and the machinery of deformation theory, we find the mathematician also involved in quite a bit of down-to-earth number-crunching. (Wiles 1995) I argue that this polysemy plays a useful role in the proof, and throws interesting light on the objectivity of the things of mathematics, as well as the growth of mathematical knowledge.